Fiber Optic Communications

Ch 1. Optical Fiber

Contents

Optical fiber as waveguide

- Maxwell's equations in an optical fiber
  - **H** is the magnetic field [A/m]

(Ampère's circuital law, no current in fiber)

$$\nabla\times\mathbf{H}=\cancel{\mathbf{J}}+\frac{\partial\mathbf{D}}{\partial t}=\frac{\partial\mathbf{D}}{\partial t}$$

  - **E** is the electric field [V/m]

(Faraday's law of induction)

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

  - **D** is the electric flux density [C/m²]

(No free electric charges in fiber)

$$\nabla\cdot\mathbf{D}=\rho=0$$

  - **B** is the magnetic flux density [V·s/m²]

(No free magnetic charges)

$$\nabla\cdot\mathbf{B}=0$$

- Relate the fields to the properties of the material
  - **P** is the **polarization field density** $\mathbf{D}=\varepsilon_0\cdot\mathbf{E}+\mathbf{P}$
    - ***The electric permittivity*** $\varepsilon_0\approx 8.85\cdot 10^{-12}\left[\mathrm{A\cdot s/V\cdot m}\right]$
  - The magnetization is zero in a fiber $\mathbf{B}=\mu_0\cdot\left(\mathbf{H}+\cancel{\mathbf{M}}\right)$
    - ***The magnetic permeability*** $\mu_0\approx 1.26\cdot 10^{-6}\left[\mathrm{A\cdot m/V\cdot s}\right]$

Wave equation

- Fourier transform equation

$$\tilde{\mathbf{E}}(\mathbf{r},\omega)=\int_{-\infty}^{\infty}\mathbf{E}(\mathbf{r},t)\cdot\exp(j\omega t)\mathrm{d}t$$

$$\mathbf{E}(\mathbf{r},t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\mathbf{E}}(\mathbf{r},\omega)\cdot\exp(-j\omega t)\mathrm{d}\omega$$

- Useful proprieties of Fourier transform

$$\mathcal{F}\left\{\frac{\partial\mathbf{E}(\mathbf{r},t)}{\partial t}\right\}=-j\omega\cdot\tilde{\mathbf{E}}(\mathbf{r},\omega)$$

$$\mathcal{F}\left\{\exp(j\Omega t)\cdot\mathbf{E}(\mathbf{r},t)\right\}=\tilde{\mathbf{E}}(\mathbf{r},\omega+j\Omega)$$

Wave equation

- The wave equation – step index fiber

$$\nabla\times\nabla\times\mathbf{E}=-\mu_0\frac{\partial}{\partial t}(\nabla\times\mathbf{H})=\mu_0\frac{\partial^2}{\partial t^2}(\varepsilon_0\mathbf{E}+\mathbf{P})=-\frac{1}{c^2}\cdot\frac{\partial^2\mathbf{E}}{\partial t^2}-\mu_0\frac{\partial^2\mathbf{E}}{\partial t^2}$$

- By using the Fourier transform of E:

$$\tilde{\mathbf{E}}(\mathbf{r},\omega)=\int_{-\infty}^{\infty}\mathbf{E}(\mathbf{r},t)\cdot\exp(j\omega t)\mathrm{d}t\ ;\ \tilde{\mathbf{P}}(\omega)=\int_{-\infty}^{\infty}\mathbf{P}(t)\cdot\exp(j\omega t)\mathrm{d}t=\varepsilon_0\tilde{\chi}(\omega)\tilde{\mathbf{E}}(\omega)$$

  - χ is the susceptibility

- The wave equation becomes

$$\nabla\times\nabla\times\tilde{\mathbf{E}}=\frac{\omega^2}{c^2}\cdot\left(1+\tilde{\chi}(\omega)\right)\tilde{\mathbf{E}}=\frac{\omega^2}{c^2}\varepsilon_r\tilde{\mathbf{E}}$$

  - ε_r is the ***relative dielectric constant*** (dimensionless)
  - The dielectric constant is related to the refractive index n and the loss α

$$\varepsilon_r=\left(n+j\frac{\alpha c}{2\omega}\right)^2$$

Wave equation

- The frequency dependence of $n(\omega)$ is referred to as material dispersion
- We use the vector rule

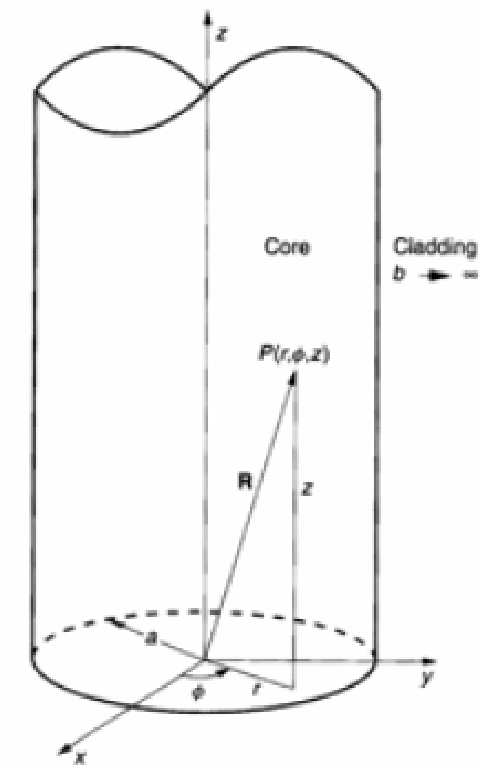
$$\nabla\times\nabla\times\tilde{\mathbf{E}}=\nabla\left(\nabla\cdot\tilde{\mathbf{E}}\right)-\nabla^2\tilde{\mathbf{E}}\approx-\nabla^2\tilde{\mathbf{E}}$$

  - It is assumed that the medium is homogeneous
- Finally the wave equation reads (using wave number $k_0=\omega/c$ and neglecting losses)

$$\nabla^2\tilde{\mathbf{E}}+n(\omega)^2\cdot k_0^2\cdot\tilde{\mathbf{E}}=0$$

Cylindrical coordinates - review

Coordinate system (r,φ,z)



$$\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial \phi}\mathbf{e}_\phi + \frac{\partial f}{\partial z}\mathbf{e}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{e}_r & \mathbf{e}_\phi & \frac{1}{r}\mathbf{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

notations:

$\mathrm{grad}\, f = \nabla f$

$\mathrm{div}\, \vec{A} = \nabla \cdot \vec{A}$

$\mathrm{rot}\, \vec{A} = \nabla \times \vec{A}$

$\mathrm{lap}\, f = \nabla^2 f$

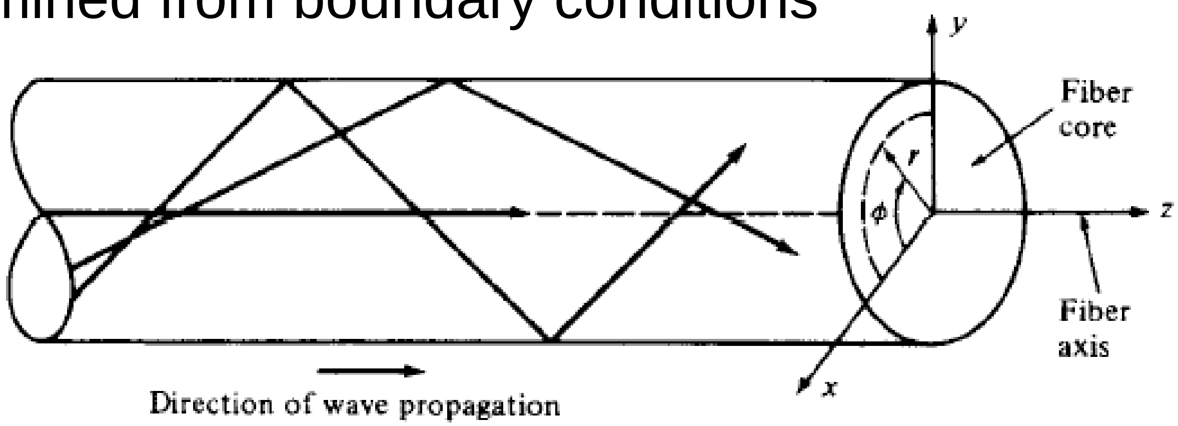
Optical fiber modes

- Solution of electromagnetic wave equation:
  - Harmonic wave – vectorial form with cylindrical coordinate

$$\begin{cases} \mathbf{E}(r,\varphi,z,t) = \mathbf{E}_0(r,\varphi)\cdot\exp\left[j(\omega\cdot t - \beta\cdot z)\right] \\ \mathbf{H}(r,\varphi,z,t) = \mathbf{H}_0(r,\varphi)\cdot\exp\left[j(\omega\cdot t - \beta\cdot z)\right] \end{cases}$$

– waves are to propagate along *z* axis:

- β(ω) is propagation constant (in direction of propagation), determined from boundary conditions



Optical fiber modes

- Boundary Conditions:
  - Tangential components of E and H continuous between core and cladding
  - If the components of E_z and H_z are not coupled together for the EM field, then the solutions for the various modes can be obtained for $E_z = 0$ or $H_z = 0$:
- $H_z = 0$, E_z # 0: transverse electric mode (TE)
- $E_z = 0$, H_z # 0: transverse magnetic mode (TM)
- The boundary conditions determine the possible values for $\beta(\omega)$

Optical fiber modes

- General solution of the wave equation in cylindrical coordinates (*r, φ, z*) :
  - Six components:

E_r , E_φ , E_z, H_r , H_φ , H_z

  - E_r , E_φ are coupled

Substitution method for transverse components (E_φ, E_ρ, H_φ, H_ρ) to solve system equations (Maxwell's eq.) with cylindrical coordinates:

$$\begin{cases} E_r = -\dfrac{j}{q^2}\left(\beta\dfrac{\partial E_z}{\partial r} + \dfrac{\mu\omega}{r}\cdot\dfrac{\partial H_z}{\partial \varphi}\right) \\ E_\varphi = -\dfrac{j}{q^2}\left(\dfrac{\beta}{r}\dfrac{\partial E_z}{\partial \varphi} - \mu\omega\cdot\dfrac{\partial H_z}{\partial r}\right) \\ H_r = -\dfrac{j}{q^2}\left(\beta\dfrac{\partial H_z}{\partial r} - \dfrac{\varepsilon\omega}{r}\cdot\dfrac{\partial E_z}{\partial \varphi}\right) \\ H_\varphi = -\dfrac{j}{q^2}\left(\dfrac{\beta}{r}\dfrac{\partial H_z}{\partial \varphi} + \varepsilon\omega\cdot\dfrac{\partial E_z}{\partial r}\right) \end{cases}$$

$$q^2 = \omega^2\mu\varepsilon - \beta^2 = k_0^2 - \beta^2$$

Optical fiber modes

- The longitudinal component E_z of wave equation:

$$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial E_z}{\partial r}\right)+\frac{1}{r^2}\cdot\frac{\partial^2 E_z}{\partial \varphi^2}+\frac{\partial^2 E_z}{\partial z^2}+n(\omega)^2\cdot k_0^2\cdot E_z=0$$

$$n(\omega)=\begin{cases} n_1(\omega) & \text{in the core } (r\leq a) \\ n_2(\omega) & \text{in the cladding } (r>a) \end{cases}$$

- The same equation is obtained for the H_z-component
- Assume that the cladding extends to infinity
- Modes are solutions of the wave equation
  - Satisfy the boundary conditions (between cladding and core)
  - Do not change spatial distribution as they propagate
  - Modes can be TE (E_z=0), TM (H_z=0) or hybrid HE or EH

Optical fiber modes

- The boundary conditions at z=0 determines whether E_z or H_z are excited
  - E_z = 0 transverse electric (TE) mode
  - H_z = 0 transverse magnetic (TM) mode
  - E_z and $H_z \neq 0$ hybrid mode, EH or HE (higher contrib to transv. fld)
- Solving the wave equation (z component) for the modes (using separation of variables method)

$$E_Z(r,\varphi,z,t)=A\cdot F_1(r)\cdot F_2(\varphi)\cdot F_3(z)\cdot F_4(t)$$

and obtain

$$F_3(z)\cdot F_4(t)=\exp\left[j(\omega t-\beta\cdot z)\right] \;\; ; \;\; F_2(\varphi)=\exp(jm\cdot\varphi)$$

- The equation for F_1 is the Bessel's equation

$$\frac{\partial^2 F_1}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial F_1}{\partial r}+\left(n^2\cdot k_0^2-\beta^2-\frac{m^2}{r^2}\right)\cdot F_1=0 \quad \text{with} \quad n^2\cdot k_0^2=\varepsilon\mu\omega^2$$

Optical fiber modes - the Bessel functions

- In the core ($r < a$, $n = n_1$), Bessel's eq. has bounded solutions J_m for $\beta < k_0 \cdot n_1 = k_1$, so that

$$F_1(r) = A \cdot J_m(p \cdot r)$$

where

$$p = \sqrt{k_1^2 - \beta^2} \;,\quad k_1 = 2\pi n_1 / \lambda$$

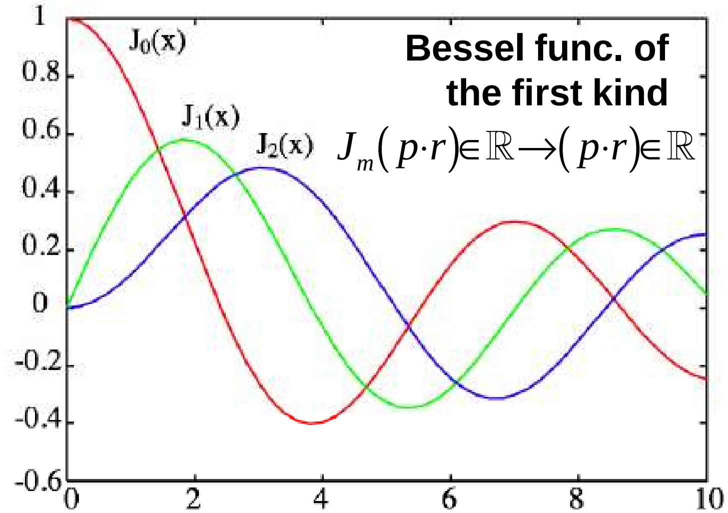
- In the cladding ($r > a$, $n = n_2$), Bessel's eq. has bounded solutions K_m for $\beta > k_0 \cdot n_2 = k_2$, so that

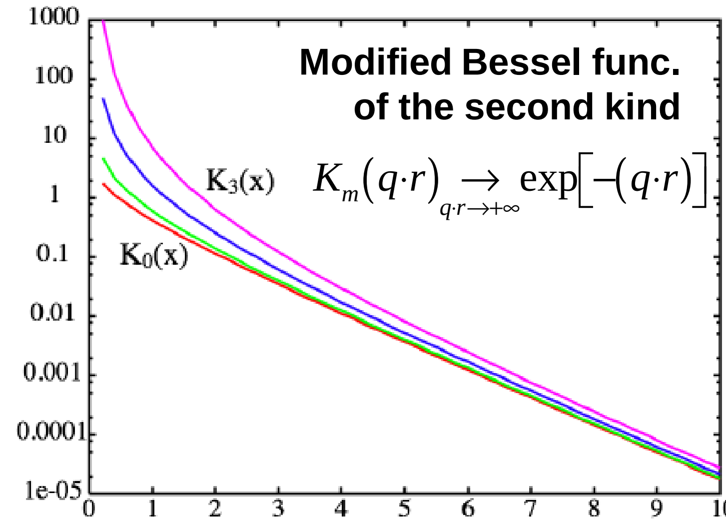
$$F_1(r) = C \cdot K_m(q \cdot r)$$

where

$$q = \sqrt{\beta^2 - k_2^2} \;,\quad k_2 = 2\pi n_2 / \lambda$$

- K_m is exponentially decaying





Optical fiber modes - Solution

Expressions for E_z and H_z are:

- core $r<a \rightarrow \left(n^2 \cdot k_0^2 - \beta^2 - m^2/r^2\right) > 0 \;\Rightarrow\; p^2 = k_1^2 - \beta^2 \;,\; k_1 = 2\pi n_1/\lambda$

$$E_z(r<a) = A \cdot J_m(p \cdot r) \cdot \exp(jm \cdot \varphi) \cdot \exp\left[j(\omega t - \beta \cdot z)\right]$$

$$H_z(r<a) = B \cdot J_m(p \cdot r) \cdot \exp(jm \cdot \varphi) \cdot \exp\left[j(\omega t - \beta \cdot z)\right]$$

- Cladding $r>a \rightarrow \left(n^2 \cdot k_0^2 - \beta^2 - m^2/r^2\right) < 0 \Rightarrow q^2 = \beta^2 - k_2^2 \;,\; k_2 = 2\pi n_2/\lambda$

$$H_z(r>a) = D \cdot K_m(q \cdot r) \cdot \exp(jm \cdot \varphi) \cdot \exp\left[j(\omega t - \beta \cdot z)\right]$$

$$E_z(r>a) = C \cdot K_m(q \cdot r) \cdot \exp(jm \cdot \varphi) \cdot \exp\left[j(\omega t - \beta \cdot z)\right]$$

where , A, B, C, D are arbitrary constants that form a system of four homogenous equations → the characteristic equation :

$$\left[\frac{J_m^{'}(p \cdot r)}{p \cdot J_m(p \cdot r)} + \frac{K_m^{'}(q \cdot r)}{q \cdot K_m(q \cdot r)}\right] \cdot \left[k_1^2 \frac{J_m^{'}(p \cdot r)}{p \cdot J_m(p \cdot r)} + k_2^2 \frac{K_m^{'}(q \cdot r)}{q \cdot K_m(q \cdot r)}\right] = \left(\frac{\beta m}{a}\right)^2 \left(\frac{1}{p^2} + \frac{1}{q^2}\right)^2$$

Optical fiber modes

- only values *p* and *q* that satisfy this equation will satisfy the boundary condition
- combining the definition of *p* and *q* we get normalized frequency relationship

$$V^2 = a^2\left(p^2 + q^2\right) = a^2\left(k_1^2 - k_2^2\right) = \left(2\pi a\right)^2 / \lambda^2 \cdot \left(n_1^2 - n_2^2\right)$$

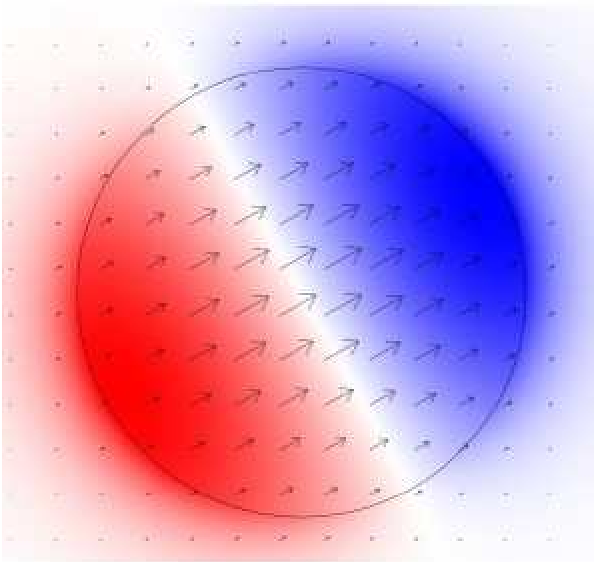
- *V* is the normalized frequency and governs the number of propagating modes
- Single mode condition : $q = 0 \;\; ; \;\; \beta = k_0 n_2$
- cutoff frequency $V < 2.405$

Optical fiber modes

- How to find the modes for a particular fiber:
  - Given the fiber geometry: the core radius ***a***, the core index n_1, and the cladding index n_2 and the operating wavelength k_0
  - For each *m*= 0, 1, 2, 3,... solve the characteristic equation to find the allowed values of β , thus determining the spatial dependence of the fields
  - For a given value of *m*, there can exist multiple solutions, which are enumerated as : *n*= 0, 1, 2, 3,...
  - For a given value of *V*, there exist solutions up to some maximum values of (*m*, *n*) (propagating modes), beyond which no solutions exist (cutoff modes)
  - For each mode (*m*, *n*), substitute *p* or *q* to find the specific propagation constant $\beta_{mn}(\omega)$
  - Each mode has a specific, wavelength-dependent propagation constant

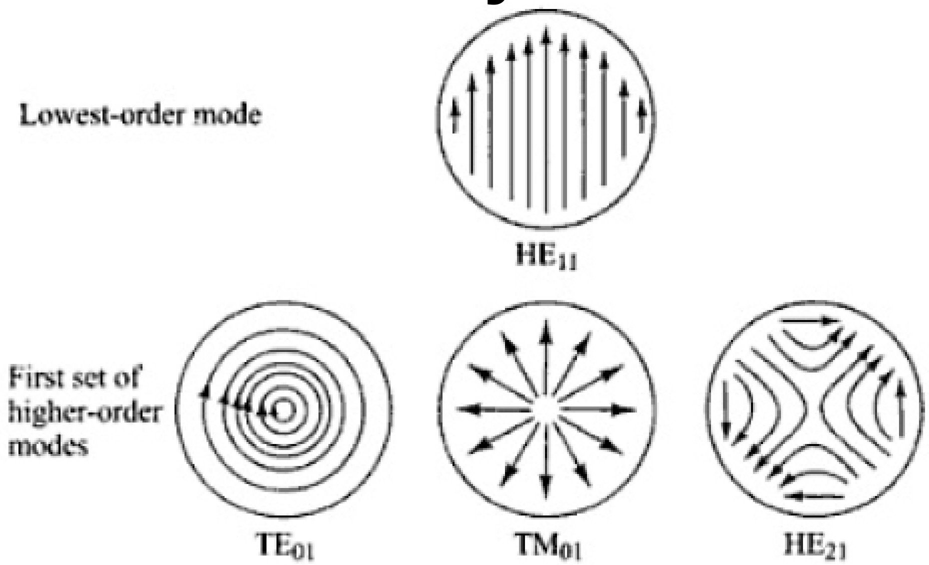
Optical fiber modes - The mode family

- For *m* = 0 either H_z or E_z are zero
  - These are called TE_{0n} and TM_{0n} modes
- The other modes are hybrid modes; EH_{mn} or HE_{mn}
- The lowest order mode is HE_{11}, which exists for all wavelengths
- A simulated HE_{11} field is seen to the right
  - E_x and E_y are shown by the arrows
  - E_z is shown by the color
- The longitudinal (z) component is much smaller than the transverse
  - Essentially linearly polarized:



Optical fiber modes - The mode family

Fiber end view al transverse electrical field vectors for 4 lowest-order modes of step index fiber



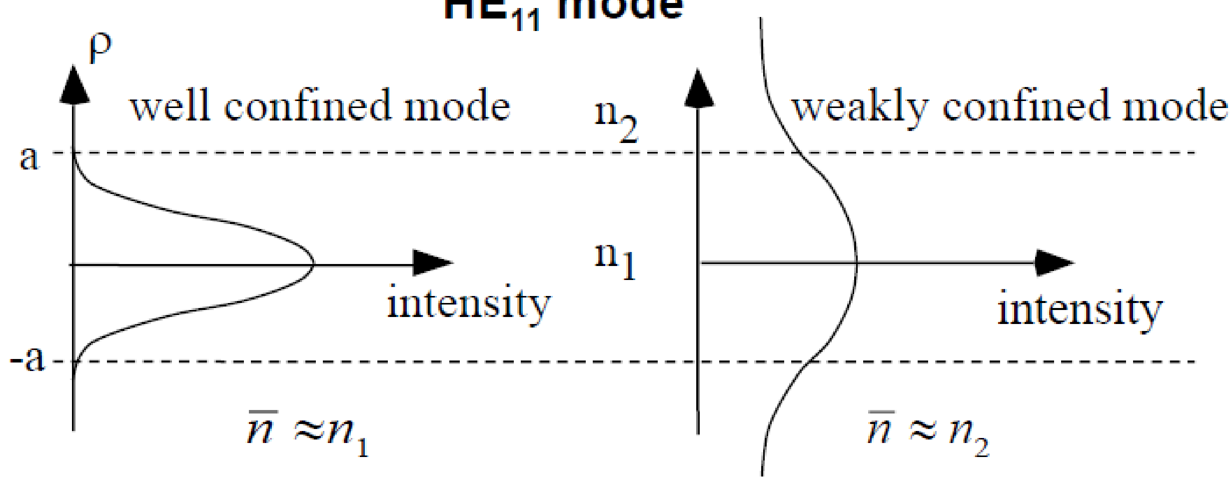
- ***Cutoff condition for Some low-order modes***

m	Modes	Cutoff condition
0	TE $_{0n}$, TH$_{0n}$	$J_{0n}(pa) = 0$
1	TE $_{1n}$, TH$_{1n}$	$J_{1n}(pa) = 0$
>2	EH$_{m,n}$	$J_{mn}(pa) = 0$
	HE$_{m,n}$	$\left(\frac{n_1^2}{n_2^2}+1\right)J_{m-1}(p\cdot a)=\frac{p\cdot a}{m-1}J_m(p\cdot a)$

The effective index (mode index)

- a mode is uniquely determined by its propagation constant β;
- the propagation constant must lie in the interval $k_0 \cdot n_2 < \beta < k_0 \cdot n_1$
- it is defined a ***mode index*** (or ***effective index***) as $\bar{n} \triangleq \beta / k_0$
- each fiber mode propagates with an effective refractive index: $\bar{n}$
- this value is between the core and cladding index: $n_2 < \bar{n} < n_1$
- the effective index gives a measure of the mode confinement :

HE$_{11}$ mode



Normalized frequency

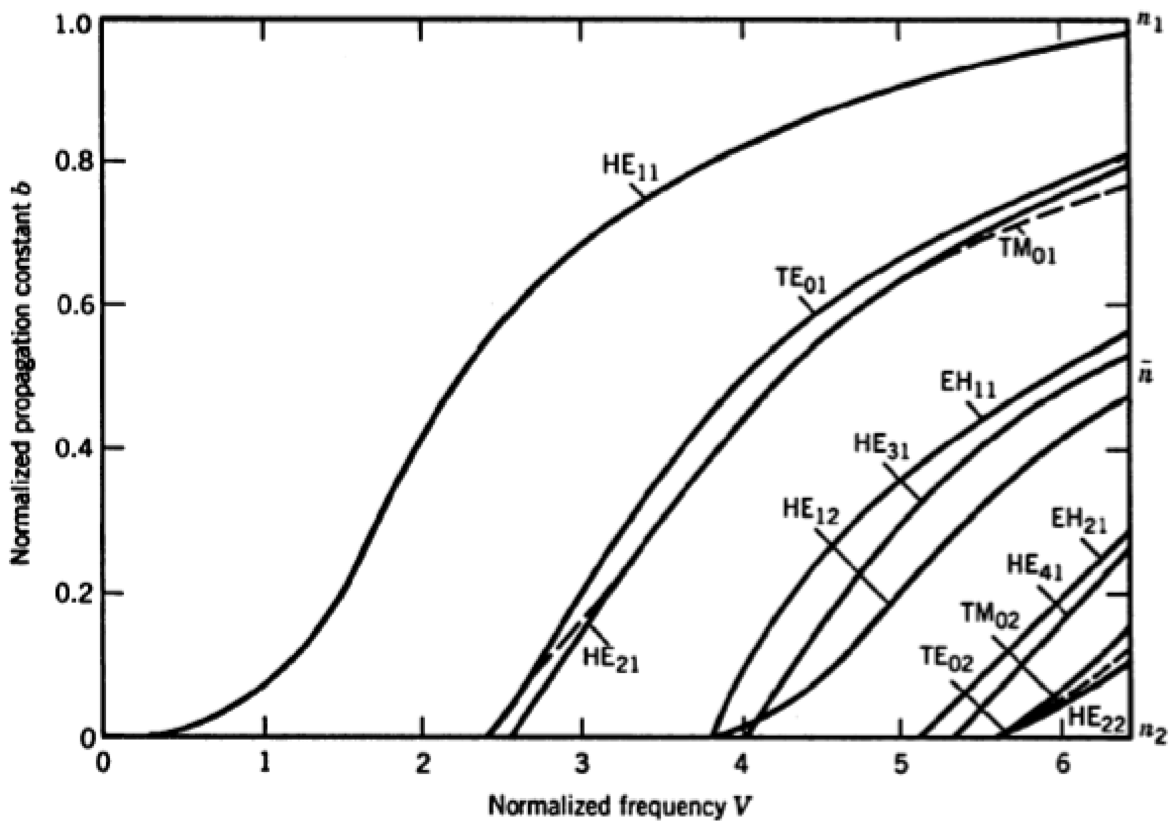
- The normalized frequency is defined $V = \frac{2\pi}{\lambda} \cdot a\sqrt{n_1^2 - n_2^2} \approx \frac{\omega}{c} \cdot a \cdot n_1 \sqrt{2\Delta}$
- The normalized propagation constant is as

$$b = \left(\bar{n} - n_2\right) / \left(n_1 - n_2\right)$$

All modes except HE_{11} are cutoff for V < 2.405.

The nr. of modes, *M*, in multimode fiber :

$$M \approx \frac{\left(n_1^2 - n_2^2\right)}{2}\left(\frac{2\pi a}{\lambda}\right) = \frac{V^2}{2}$$

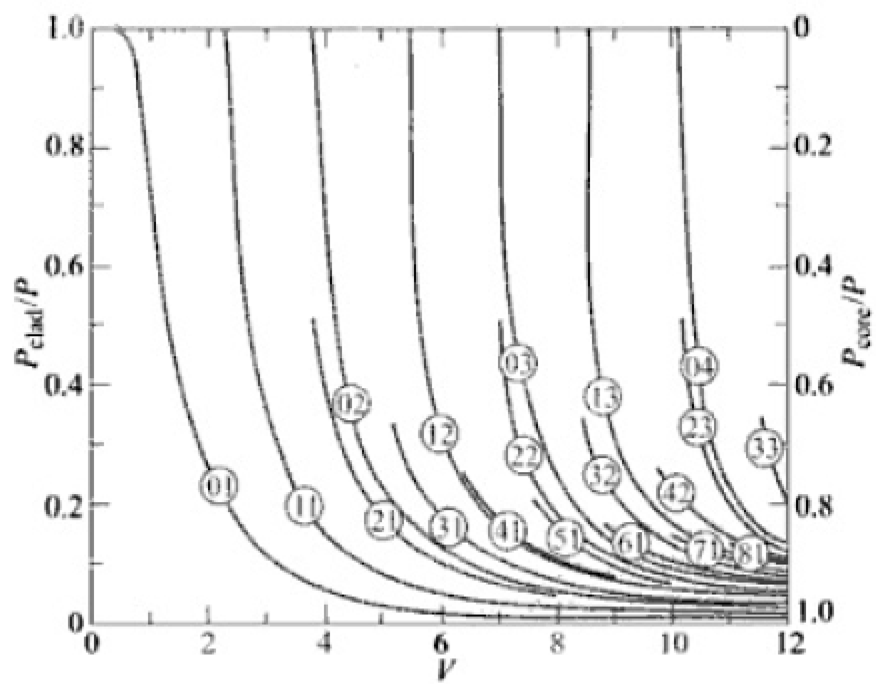


Power distribution - multimode fiber step index

- Important parameters: core diameter and numeric aperture
- P_{core} increase with normalized wavelength
- $P=P_{core} + P_{clad}$

$$\frac{P_{core}}{P} = 1 - \frac{P_{clad}}{P}$$

$$\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}$$



Graded index Fiber structure

- Variable refractive index
  - *r* radial distance from fiber axis
  - *a* core radius
  - n_1 *refractive index of core*
  - n_2 *refractive index of cladding*
  - *α* shape of index profile
  - *Δ the index difference*

$$n(r)=\begin{cases} n_1\sqrt{1-2\Delta\left(\frac{r}{a}\right)^{\alpha}} & \text{where } 0\le r\le a \\ n_2=n_1\sqrt{1-2\Delta}\approx n_1(1-\Delta) & \text{where } a\le r \end{cases}$$

$$\Delta=\frac{n_1^2-n_2^2}{2n_1^2}\approx\frac{n_1-n_2}{n_1}$$

- Variable NA

$$NA(r)=\begin{cases} \sqrt{n^2(r)-n_2^2}=NA(0)\sqrt{1-\left(\frac{r}{a}\right)^{\alpha}} & \text{when } 0\le r\le a \\ 0 & \text{when } a\le r \end{cases}$$

$$NA(0)=\sqrt{n^2(0)-n_2^2}\approx n_1\sqrt{2\Delta}$$

- The number of modes for graded index fiber

$$M=\frac{\alpha}{\alpha+2}a^2k^2n_1^2\Delta$$

Summary MMF

Guided Modes

- *total internal reflection*
- additional phase condition

Mode theory

- *E* is not totally confined in the guide
- *E decays exponentially outside the guide*
- *E is stationary (sinusoidal variation) in the transversal direction within the guide*
- Higher-order modes penetrate more in the cladding
- Solution of the wave equation given by Bessel functions
- β takes discrete values defined by the boundary conditions (determined by the characteristic equation)
- Condition for single-mode operation ($V<2.405$)

Single Mod Fiber (SMF)

- Single-mode condition $\rightarrow$ the value of V at which the TE_{01} and TM_{01} modes reach cutoff ($J_0(V) = 0 \rightarrow V < 2.405$)
- Core radius can be estimated with: $a = \frac{V \cdot \lambda}{2\pi\sqrt{n_1^2 - n_2^2}} \approx \frac{V \cdot c}{\omega \cdot n_1 \sqrt{2\Delta}}$
- Supports only the fundamental HE_{11} mode;
- The mode index: $\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta)$
- Approximation of normalized propagation constant for V=1.5$\div$ 2.5:

$$b(V) \approx (1{,}1428 - 0.9960/V)^2$$

- SMF filed characteristics (for $\Delta \ll 1$)
  - *z*-components of the E/H fields (HE_{11} mode) are quite small
  - corresponds to meridional rays
  - E_x or E_y component is dominating and are independent of φ
  - This mode is essentially linearly polarized ($HE_{11} \equiv LP_{01}$)

Single Mod Fiber (SMF)

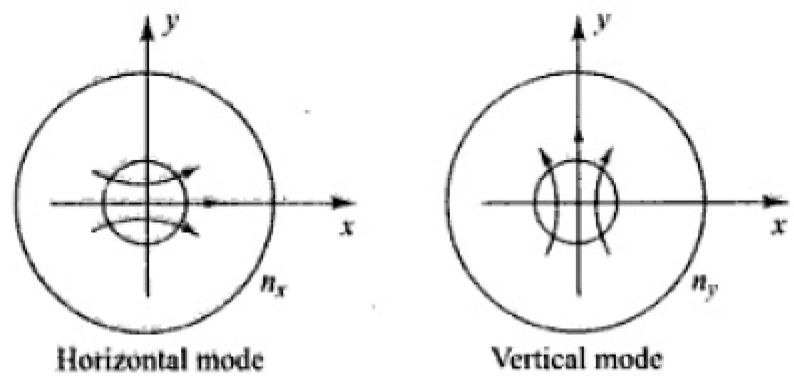
- For transversal evaluation component, using (*x, y, z*) coordinates, setting: $E_y = 0 \leftrightarrow H_x = 0$ (mode linearly polarized along the x axis):

$$E_x = \begin{cases} E_0 \cdot \left(J_0(p \cdot r) / J_0(p \cdot a)\right) \cdot \exp(j\beta \cdot z) & ; \quad (r \leq a) \\ E_0 \cdot \left(K_0(p \cdot r) / K_0(p \cdot a)\right) \cdot \exp(j\beta \cdot z) & ; \quad (r > a) \end{cases}$$

$$H_y = n_2 \sqrt{\varepsilon_0 / \mu_0} \cdot E_x$$

$$E_y = H_x = 0$$

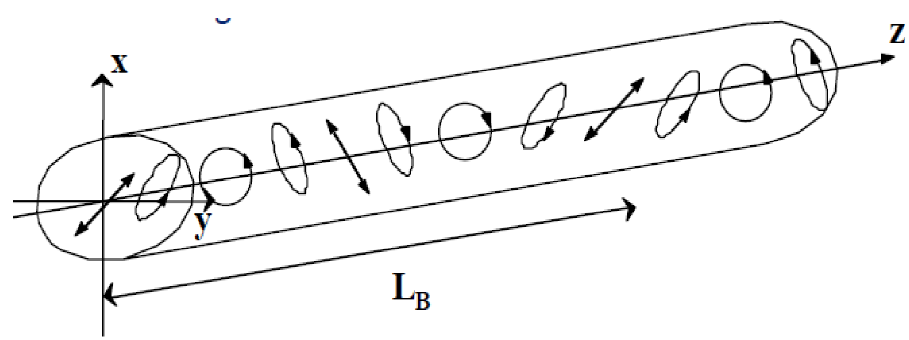
- The same fiber support another mode linearly polarized along the y axis ($E_x = 0 \leftrightarrow H_y = 0$).



- SMF supports two orthogonally polarized mode, on X and Y axis, and have the same mode index (*m*=0) .

SMF - Fiber Birefringence

- an ideal single-mode fiber with a perfectly cylindrical core of uniform diameter has orthogonally polarized modes (LP)
- real fibers have variations in the core shape and experience nonuniform stress such that the cylindrical symmetry is broken.
  - The LP-mode degeneracy is removed
  - The fiber is birefringent with a “slow” and a “fast” axis (*x* or *y*)
  - The degree of modal birefringence is defined by: $B_m = |\bar{n}_x - \bar{n}_y|$
  - Birefringence leads to a periodic power exchange between the two linear polarized components
  - The period, (*the beat length*), is given by $L_B = \lambda / B_m$

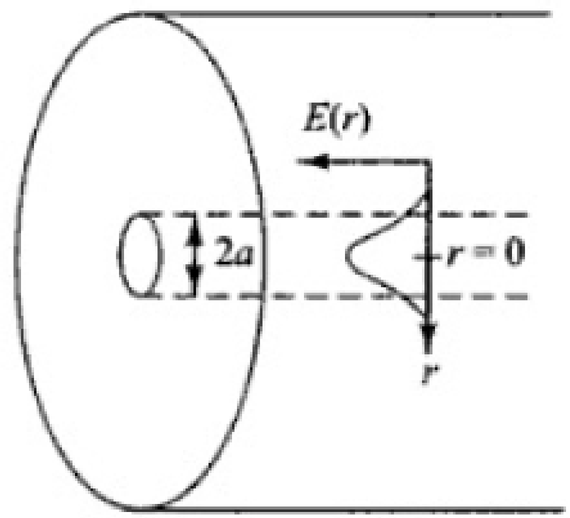


The state of polarization changes along the fiber length from linear to elliptical, and then back to linear (periodically).

SMF – spot size (Mode Filed Diameter – MFD)

- The amplitude distribution of E for HE_{11} mode in the transversal plane is: $E(r)=E_0\cdot\left(J_0(p\cdot r)/J_0(p\cdot a)\right)\cong E_0\exp\left(-r^2/w_0^2\right)$

- It is approximated with *Gaussian* shape
- The fraction of the power contained in the core can be obtained with



$$\Gamma=\frac{P_{core}}{P_{total}}=\frac{\int_0^a E_x^2(r)\cdot r\mathrm{d}r}{\int_0^\infty E_x^2(r)\cdot r\mathrm{d}r}=1-\exp\left(-\frac{2a^2}{w_0^2}\right)$$

- Exp:

$$\text{for } V=2 \;\rightarrow \Gamma\cong 75\%$$
$$\text{for } V=1 \;\rightarrow \Gamma\cong 20\%$$

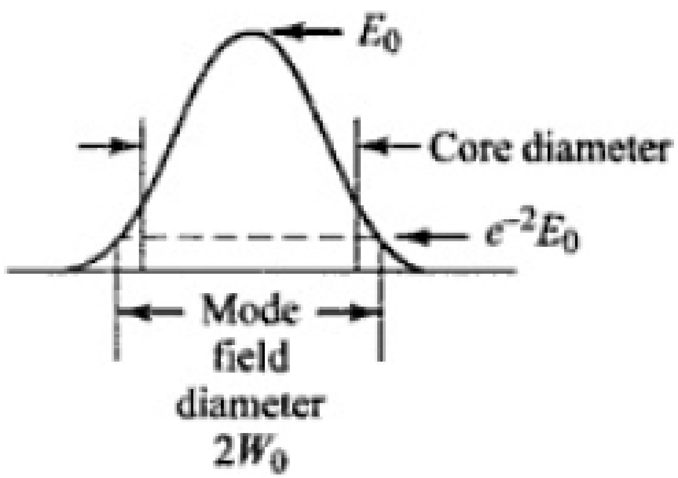
SMF – spot size (Mode Filed Diameter – MFD)

- spot size (MFD) is diameter ($2w_0$) of amplitude distribution at a level e^{-2} (13.53%) from the peak

$$MFD = 2w_0 = 2\sqrt{\int_0^\infty E^2(r)\cdot r^3 \mathrm{d}r \Big/ \int_0^\infty E^2(r)\cdot r\mathrm{d}r}$$

- Approximation of MFD for 1.5 < V < 2.5

$$\frac{w_0}{a} \approx 0.65 + 1.619\cdot V^{-\frac{3}{2}} + 2.879\cdot V^{-6}$$



- most telecommunication single-mode fibers are designed to operate in the range 2<V<2.4

SMF – Dispersion

- Intermodal Dispersion is absent in SMF because the energy of the injected pulse is transported by a single mode.

(intermodal dispersion is explain as different paths followed by different rays in the geometrical-optics description and as different mode indices associated with different modes, in the wave theory)

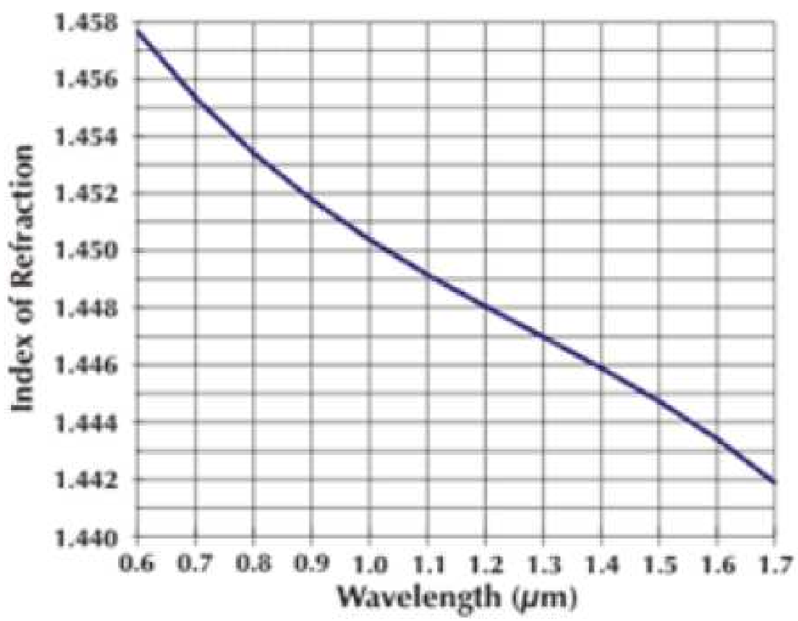
- ***Group Velocity Dispersion*** (*D*) group velocity is different for different frequency components.
  - ***material dispersion*** D_M (the dependence of *n* on *ω*)
  - ***waveguide dispersion*** D_W (the mode behavior, which makes *β* depend on *ω*)

$$D = D_W + D_M$$

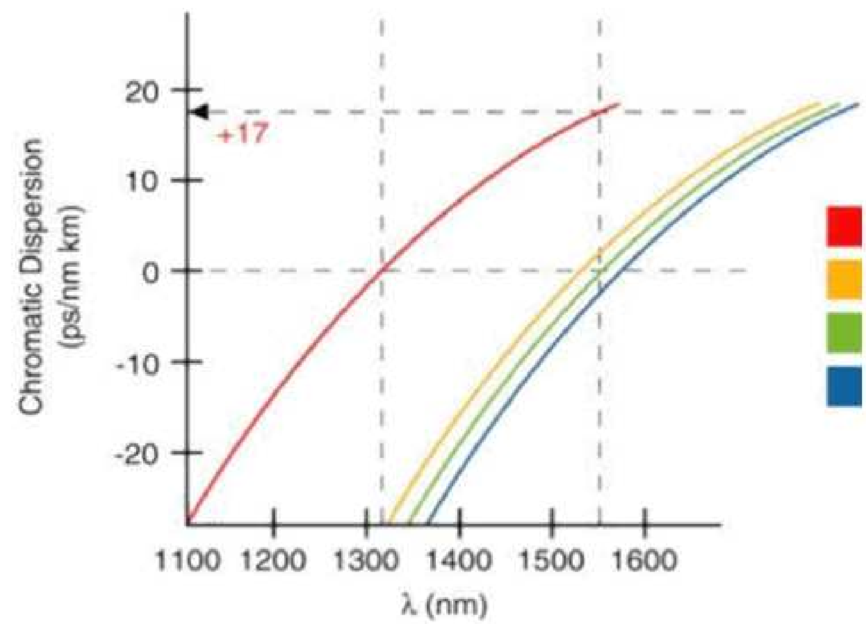
- ***Higher-Order Dispersion***
- ***Polarization-mode dispersion***

SMF - Dispersion, qualitatively

- Different wavelengths (frequency components) propagate differently (with different speed)
- A pulse has a certain spectral width and will broaden during propagation



The index of refraction as a function of wavelength



The dispersion in SMF (red) and different dispersion-shifted fibers

Propagating waves, phase velocity

- The electric field in (*x*,*y*,*z*) coordinates is written as

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\left\{\hat{\mathbf{x}}\cdot E_{XY}(x,y)\cdot A(z,t)\cdot\exp(j\beta_0 z-j\omega_0 t)\right\}$$

  - The field is polarized in the **x**-direction
  - $E_{XY}(x, y)$ describes the mode in the transverse directions
  - $A(z, t)$ is the complex ***field envelope***
  - β_0 is the propagation constant corresponding to ω_0

Each frequency component ω of the $A(z, t)$ propagates according to (in the Fourier domain): $\tilde{A}(z,\omega)=\tilde{A}(0,\omega)\cdot\exp(j\beta z)$

$$\left(\tilde{A}(z,\omega)=\int_{-\infty}^{+\infty} A(z,t)\cdot\exp(j\omega t)\mathrm{d}\omega\right) \quad \text{where } \tilde{A}(0,\omega)=\mathcal{F}\left\{A(0,t)\right\}$$

Each spectral component of a pulse propagates differently.

Propagating waves, phase velocity

- Shows that the power spectrum, which is proportional to $|E(\omega)|^2$, will not change during transmission for specific *z.*

- The propagation constant, β, is complex:

$$\beta(\omega)=\frac{\overline{n}(\omega)+\delta n_{NL}(\omega)}{\omega/c}+j\frac{\alpha(\omega)}{2}\approx\beta_L(\omega)+\beta_{NL}(\omega)+j\frac{\alpha(\omega)}{2}$$

  - α is the attenuation
  - δn_{NL} is a small nonlinear, power dependent change of the refractive index
  -

- Dispersion arises from $\beta_L(\omega)$
  - the frequency dependence of β_{NL} and α is small

Phase and group velocity

- For pulses with $\Delta\omega << \omega_0$ (quasi-monochromatic approximation) expand $\beta_L(\omega)$ in a Taylor series around the carrier frequency ω_0

with notations: $\Delta\omega = \omega - \omega_0$ and $\beta_m = \left.\frac{d^m \beta_L}{d\omega^m}\right|_{\omega=\omega_0}$

$$\beta_L(\omega) = \bar{n}(\omega)\frac{\omega}{c} = \beta_0 + \beta_1 \cdot \Delta\omega + \tfrac{1}{2}\cdot\beta_2\cdot(\Delta\omega)^2 + \tfrac{1}{6}\cdot\beta_3\cdot(\Delta\omega)^3 + \ldots,$$

$\beta_0 \to 1/v_p$, $\beta_1 \to 1/v_g$, $\beta_2 \to$ *GVD*, $\beta_3 \to$ Dispersion slope (rel. to S)

- A monochromatic wave propagates with the phase velocity

$$v_p = \frac{\omega_0}{\beta_0}$$

Phase and group velocity

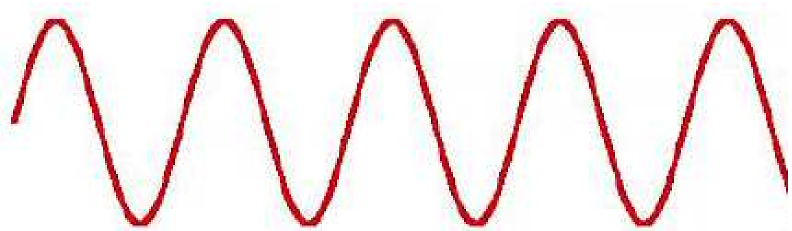
- The propagating field is

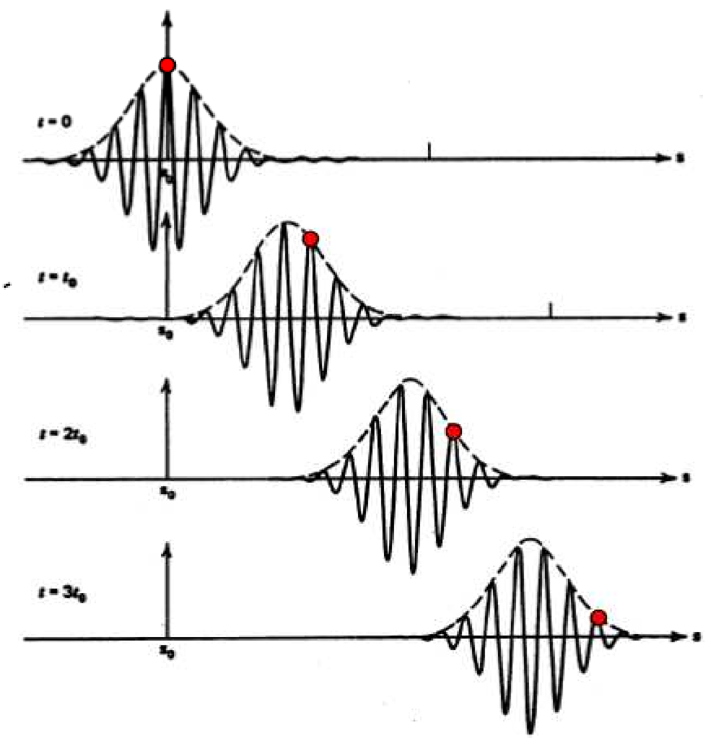
$$E(z,t)=E_0\cdot\cos(\beta_0 z-\omega_0 t)$$

- The phase velocity is $v_p=\omega_0/\beta_0$
- A pulse E(t) at z = 0 will propagate as

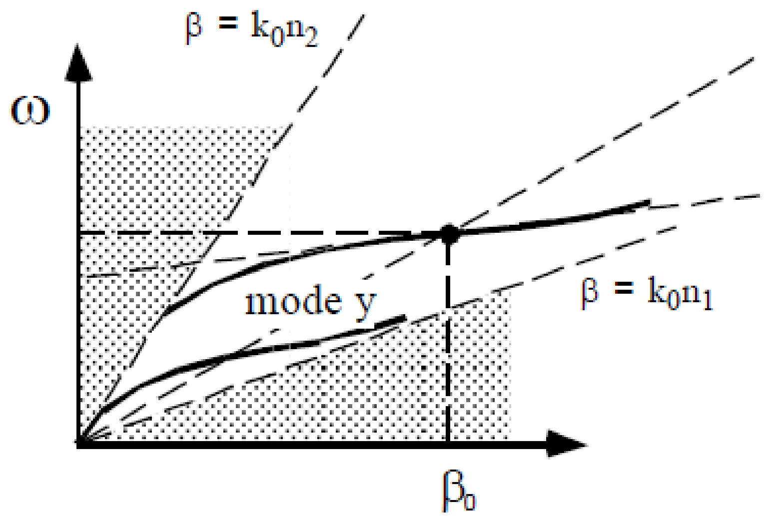
$$E(z,t)=E\left(\frac{\mathrm{d}\beta}{\mathrm{d}\omega}z-t\right)$$

- i.e. it moves with the group velocity





$$\frac{1}{v_g}=\left.\frac{\mathrm{d}\beta}{\mathrm{d}\omega}\right|_{\omega=\omega_0}$$



SMF – Group Velocity Dispersion (GVD)

- Each frequency component has its *group velocity* v_g

$$v_g = 1/(\mathrm{d}\beta/\mathrm{d}\omega) = c/\bar{n}_g$$

where n_g is the group index given by: $\bar{n}_g = \bar{n} + \omega \cdot (\mathrm{d}\bar{n}/\mathrm{d}\omega)$

- For $\Delta\omega$ spectral width of the pulse, the extent of pulse broadening for a fiber of length *L* is:

$$\Delta T = \frac{\mathrm{d}T}{\mathrm{d}\omega}\cdot\Delta\omega = \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{L}{v_g}\right)\cdot\Delta\omega = L\frac{\mathrm{d}^2\beta}{\mathrm{d}\omega^2}\cdot\Delta\omega = L\beta_2\cdot\Delta\omega = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{L}{v_g}\right)\cdot\Delta\lambda = DL\cdot\Delta\lambda$$

where GVD parameter $\beta_2 = \mathrm{d}^2\beta/\mathrm{d}\omega^2$

- *D* is called the dispersion parameter and is expressed in units of ps/(km*nm)

$$D = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2$$

SMF – Group Velocity Dispersion (GVD)

Each spectral component of a pulse has a specific group velocity

The ***group delay*** after a distance *L* is

$$T = \frac{L}{v_g} = L\frac{\mathrm{d}\beta}{\mathrm{d}\omega}$$

The group velocity is related to the ***mode group index*** given by

$$v_g = \frac{1}{\frac{\mathrm{d}\beta}{\mathrm{d}\omega}} = \frac{c}{\bar{n}_g} = \frac{c}{\bar{n} + \omega\frac{\mathrm{d}\bar{n}}{\mathrm{d}\omega}} = \frac{c}{\bar{n} - \lambda\frac{\mathrm{d}\bar{n}}{\mathrm{d}\lambda}}$$

Assuming the spectral width $\Delta\omega$, the time pulse broadening is

$$\Delta T = \frac{\mathrm{d}T}{\mathrm{d}\omega}\cdot\Delta\omega = \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{L}{v_g}\right)\cdot\Delta\omega = L\frac{\mathrm{d}^2\beta}{\mathrm{d}\omega^2}\cdot\Delta\omega = L\beta_2\cdot\Delta\omega \qquad \beta_2 = \frac{\mathrm{d}^2\beta}{\mathrm{d}\omega^2}$$

where β_2 is known as the ***GVD parameter*** (unit is s^2/m or ps^2/km)

SMF – Group Velocity Dispersion (GVD)

Measuring the spectral width in units of wavelength (rather than frequency), we can write the broadening as:

$$\Delta T = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{L}{v_g}\right)\cdot\Delta\lambda = D\cdot\Delta\lambda\cdot L$$

where D [ps/(nm·km)] is called the ***dispersion parameter***

D is related to β_2 and the effective mode index according to:

$$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{2\pi c}{\lambda^2}\cdot\frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{1}{v_g}\right) = -\frac{2\pi}{\lambda^2}\cdot\left(2\frac{\mathrm{d}\overline{n}}{\mathrm{d}\omega} + \omega\cdot\frac{\mathrm{d}^2\overline{n}}{\mathrm{d}\omega^2}\right)$$

The dispersion parameter has two contributions:

***material dispersion*, DM**: The index of refraction of the fiber material depends on the frequency

***waveguide dispersion*, DW**: The guided mode has a frequency dependence

SMF – Group Velocity Dispersion (GVD)

The total dispersion *D* is the sum of the waveguide and material contributions:

$$D = D_M + D_W$$

An estimate of the dispersion limited bit-rate is:

$$|D| \cdot B \cdot \Delta\lambda \cdot L < 0.5$$

where - B is the bit-rate,

- Δλ the spectral width

- L the fiber length

SMF – Material Dispersion

- Material dispersion occurs because the refractive index of silica changes with the optical frequency. It is related to the characteristic resonance frequencies at which the material absorbs the electromagnetic radiation

- It is well approximated by the *Sellmeier equation* :

$$n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}$$

where ω_j is the resonance frequency , B_j is the oscillator strength

For silica: B_1=0.6961663, B_2=0.4079426, B_3=0.8974794

λ_1=0.0684043 μm, λ_2=0.1162414 μm, λ_3=9.896161 μm

Material dispersion:

$$D_M = \frac{2\pi}{\lambda^2} \cdot \frac{\mathrm{d}n_{2g}}{\mathrm{d}\omega} = \frac{1}{c} \cdot \frac{\mathrm{d}n_{2g}}{\mathrm{d}\lambda}$$

n_{2g} is the group index of the cladding material

SMF – Material Dispersion

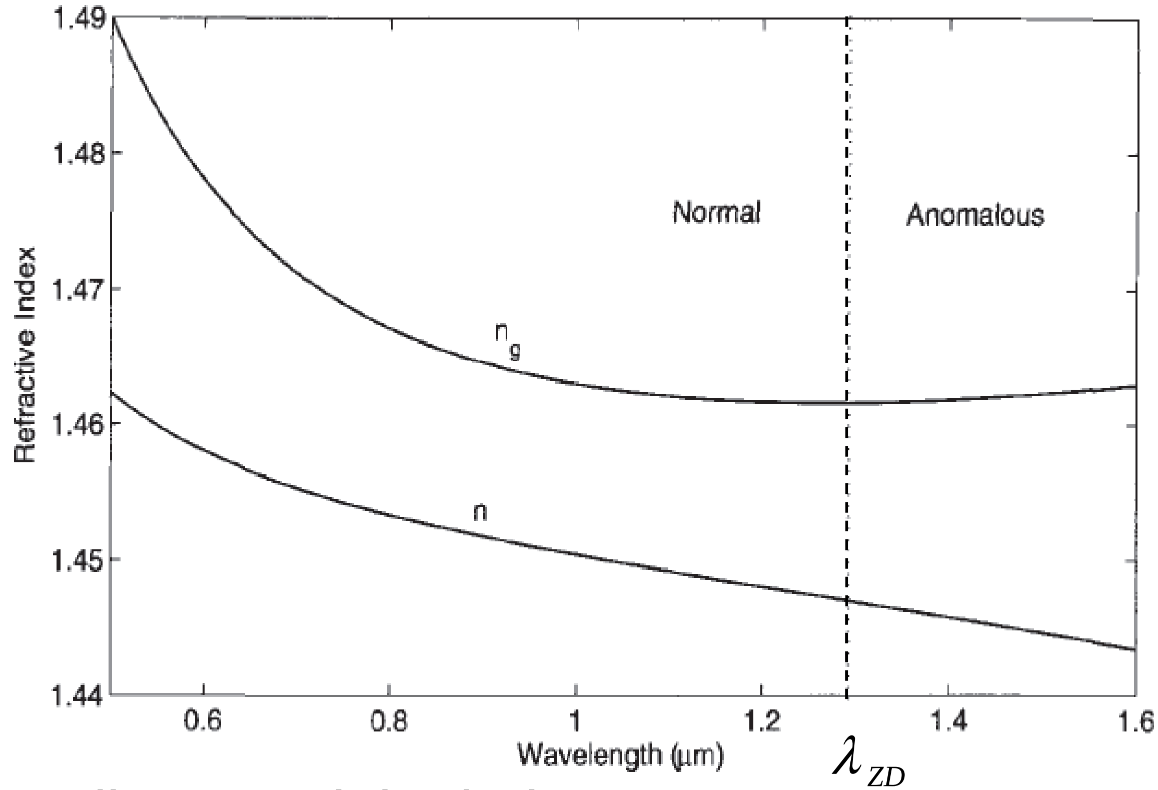
- approximation for $\lambda \in [\,1.25\text{-}1.66\,]$ μm

$$D_M \approx 122 \cdot \left(1 - \frac{\lambda_{ZD}}{\lambda}\right)$$

- λ_{ZD} *zero-dispersion wavelength.*
- λ_{ZD} =1.27 – 1.31 μm

for pure silica:

λ_{ZD} =1.276 μm



λ_{ZD} depends on the core radius *a* and the index step Δ

The dispersion parameter D_M is negative below λ_{ZD} (normal dispersion) and positive above that (anomalous dispersion).

SMF - Waveguide dispersion

The waveguide dispersion arises from the modes' dependence (V) on frequency

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\cdot\left[\frac{n_{2g}^2}{n_2\omega}\cdot\frac{V\cdot \mathrm{d}^2(Vb)}{\mathrm{d}V^2}+\frac{\mathrm{d}n_{2g}}{\mathrm{d}\omega}\cdot\frac{\mathrm{d}(Vb)}{\mathrm{d}V}\right]$$

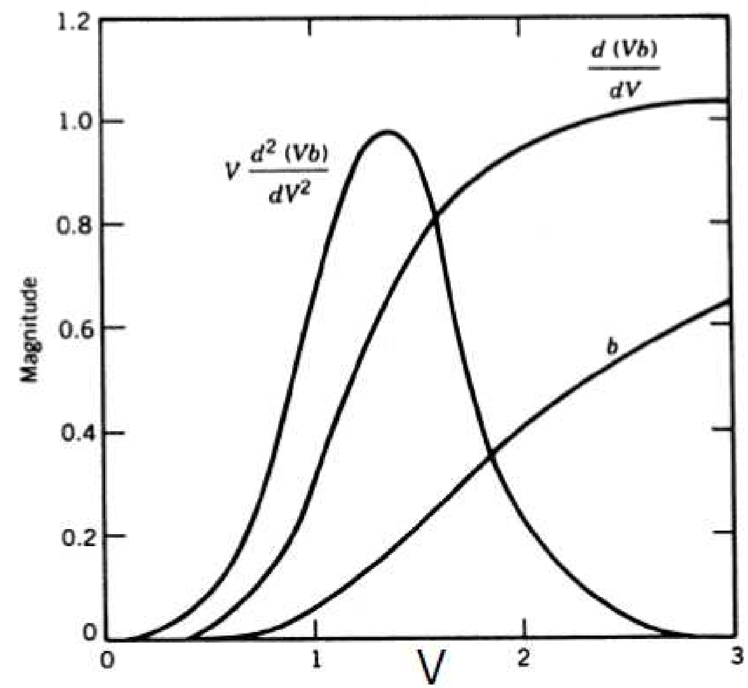
n_{2g}: the cladding group index

V: the normalized frequency

$$V = \frac{2\pi}{\lambda}a\sqrt{n_1^2 - n_2^2} \approx \frac{\omega}{c}an_1\sqrt{2\Delta}$$

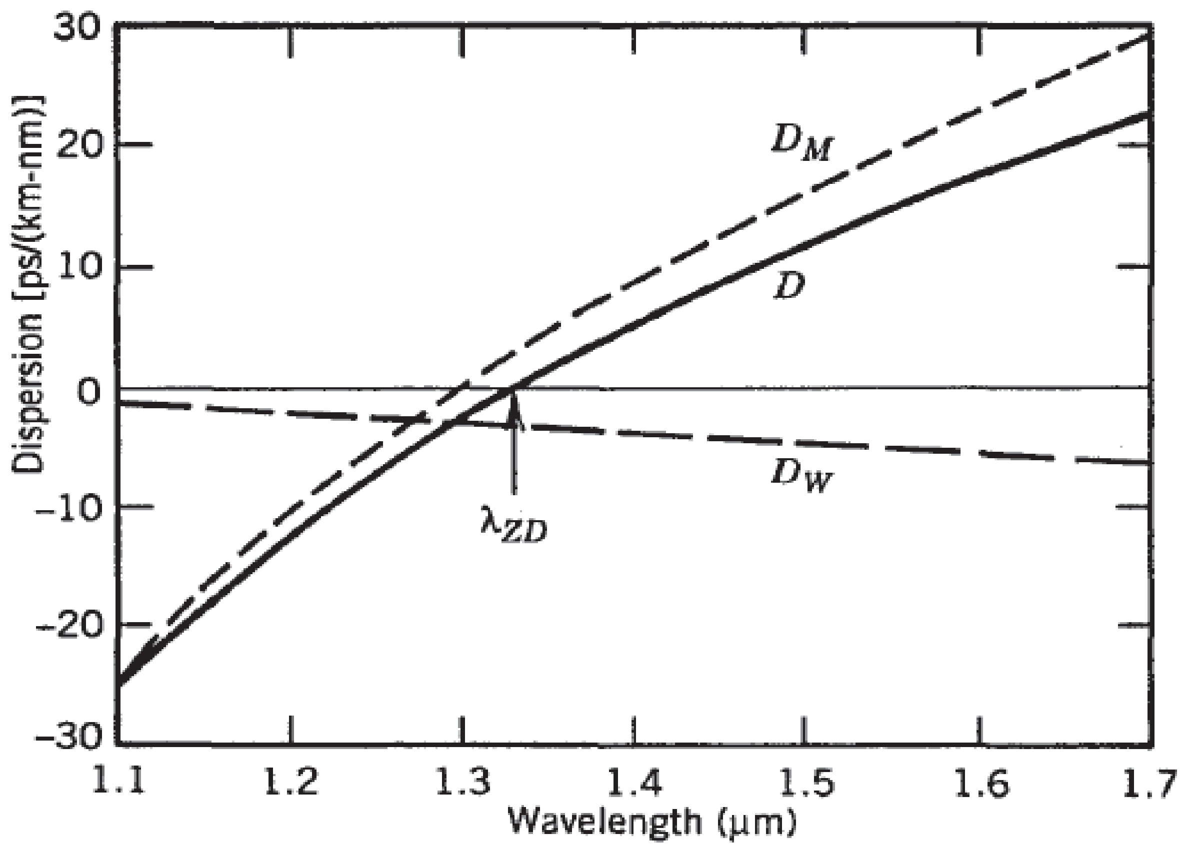
b: the normalized waveguide index

$$b = \frac{\bar{n} - n_2}{n_1 - n_2}$$



SMF – Waveguide Dispersion

- D_W is negative in the wavelength range 0-1.6µm
- D_W depends on the core radius *a* and the index step Δ.



SMF – GVD dispersion management

- adjust D_W for

-dispersion-shifted fibers

$$D = D_W + D_M = 0$$

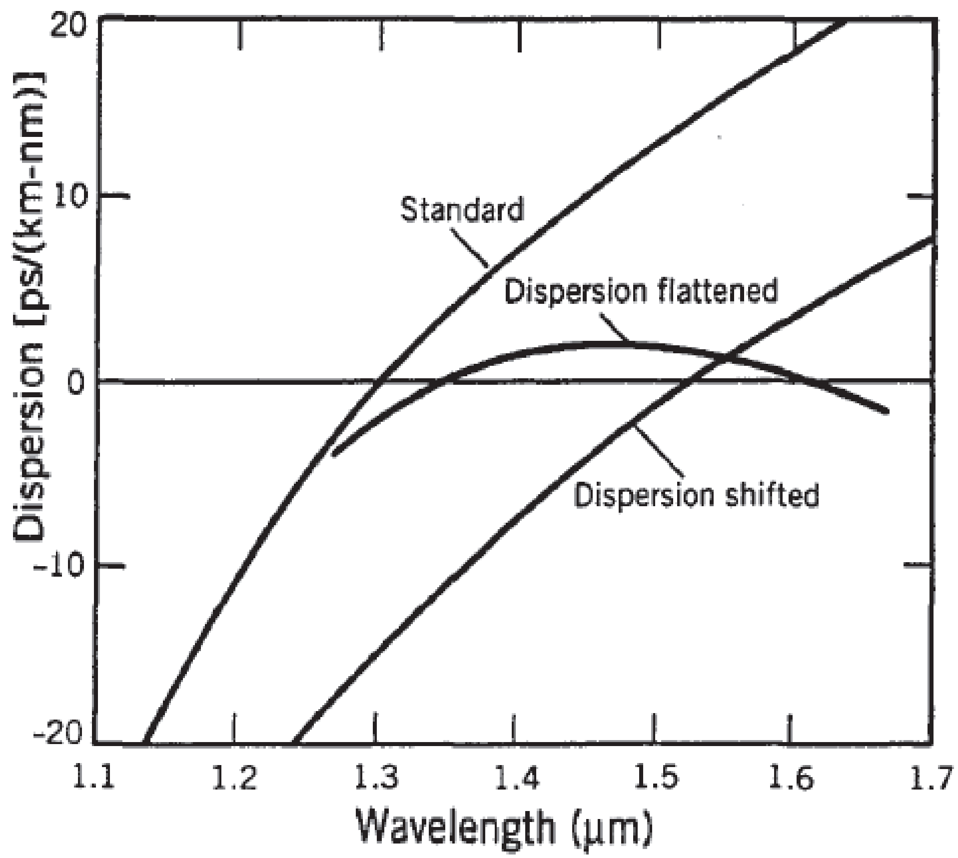
$$\text{for} \quad \lambda_{ZD} \approx 1.55\mu\text{m}$$

- dispersion-flattened fibers

for λ in range of 1.3 to 1.6 μm (used for telecommunications systems, due small *optical loss* – 1.55 μm).

- dispersion-decreasing fibers

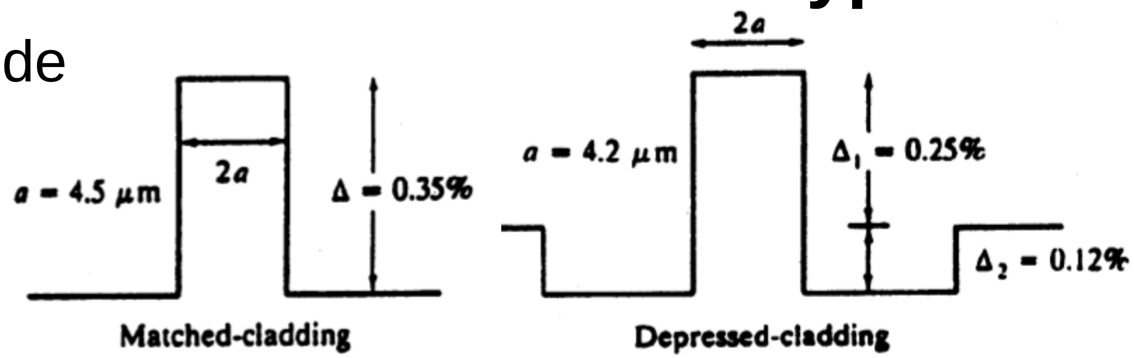
where GVD decreases along the fiber length.



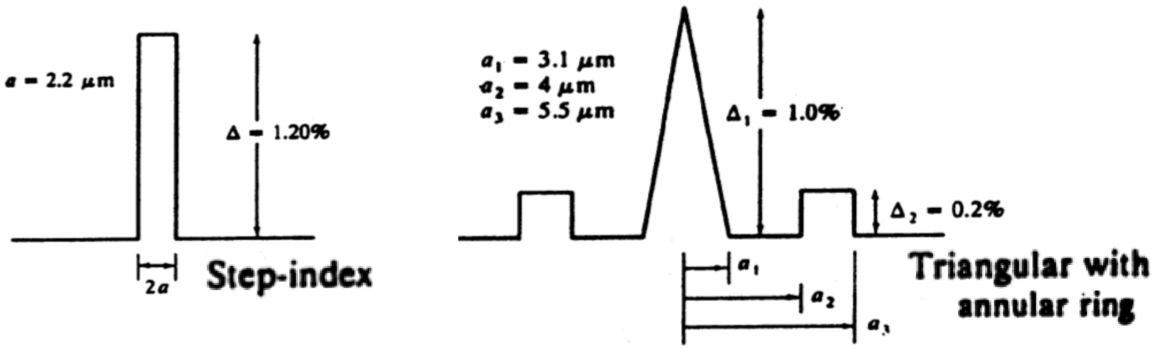
- dispersion-compensating fibers, GVD is made normal and has a relatively large magnitude

SMF - Index profiles of different fiber types

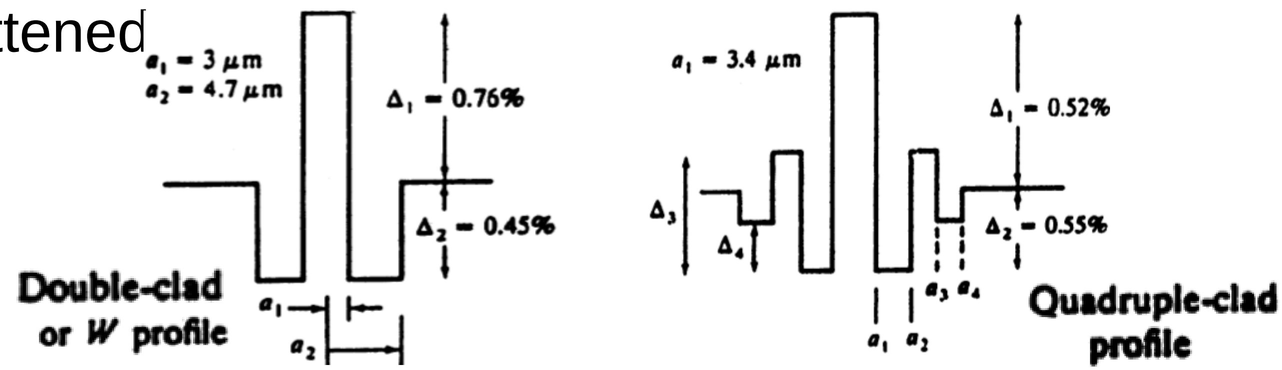
- Standard single-mode fiber (SMF)



- Dispersion - shifted Fiber (DSF)



- Dispersion - flattened Fiber (DFF)



SMF – Higher-Order Dispersion

The dispersive effects do not disappear completely at $\lambda=\lambda_{ZD}$, (D=0)

Because D cannot be made zero at all wavelengths within the pulse spectrum centered at λ_{ZD} → ***Higher-Order Dispersion.***

- ***Differential-dispersion*** (*dispersion slope*) , S :

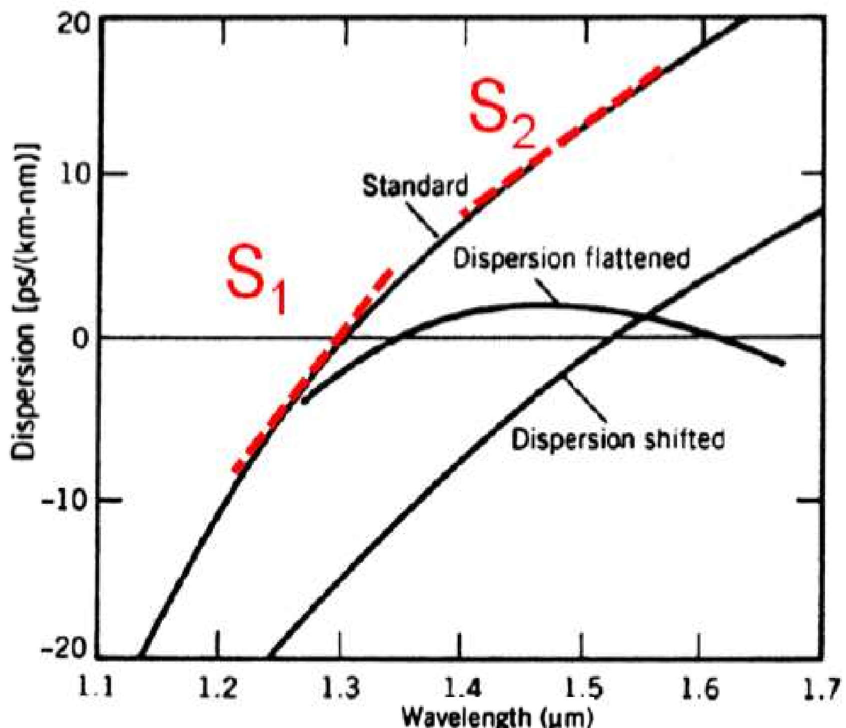
$$S=\frac{\mathrm{d}D}{\mathrm{d}\lambda}=\left(\frac{2\pi c}{\lambda^2}\right)^2\cdot\beta_3+\frac{4\pi c}{\lambda^3}\cdot\beta_2$$

$$\cong\left(\frac{2\pi c}{\lambda^2}\right)^2\cdot\beta_3 \quad ; \quad \beta_j=\frac{\mathrm{d}^j\beta}{\mathrm{d}\omega^j}$$

- Typical value in SMF is 0.07 ps/(nm² ·km).

- Important in the WDM system

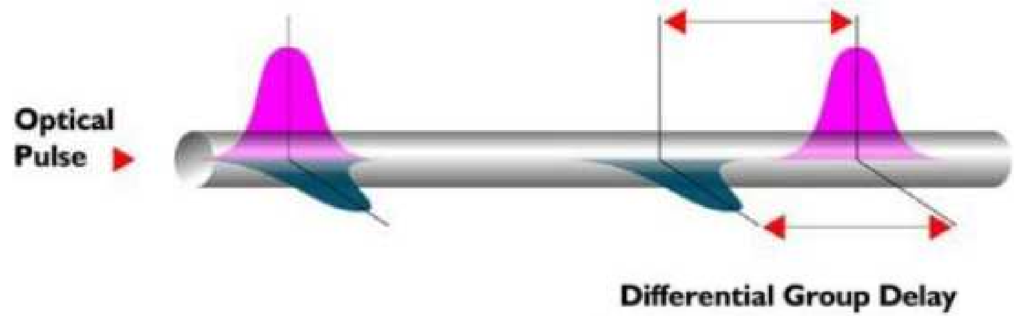
$$D\big|_{\lambda_{ZD}}=S\cdot\Delta\lambda \;\rightarrow\; |S|\cdot B\cdot\Delta\lambda^2\cdot L<0.5$$



Polarization-mode dispersion (PMD)

An optical fiber is always slightly elliptical:

- Different index of refraction for x and y polarization, difference= Δn
  - Changes along the fiber
  - Changes with frequency
  - Changes with time



This leads to:

- The state-of-polarization (SOP) of the signal in the fiber is random
- Different delay for different polarizations,
- ***differential group delay*** (DGD)

$$\Delta\tau = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| = L\left|\beta_{1x} - \beta_{1y}\right| = L \cdot \Delta\beta_1$$

- DGD acting along varying axes leads to ***polarization-mode dispersion*** (PMD)

- *Dp* is the PMD parameter

(typical value is 0.05 –1 ps/√km)

$$\sigma_T \approx (\Delta\beta_1)\sqrt{2 l_c z} = D_p \sqrt{L}$$

Polarization-preserving fibers

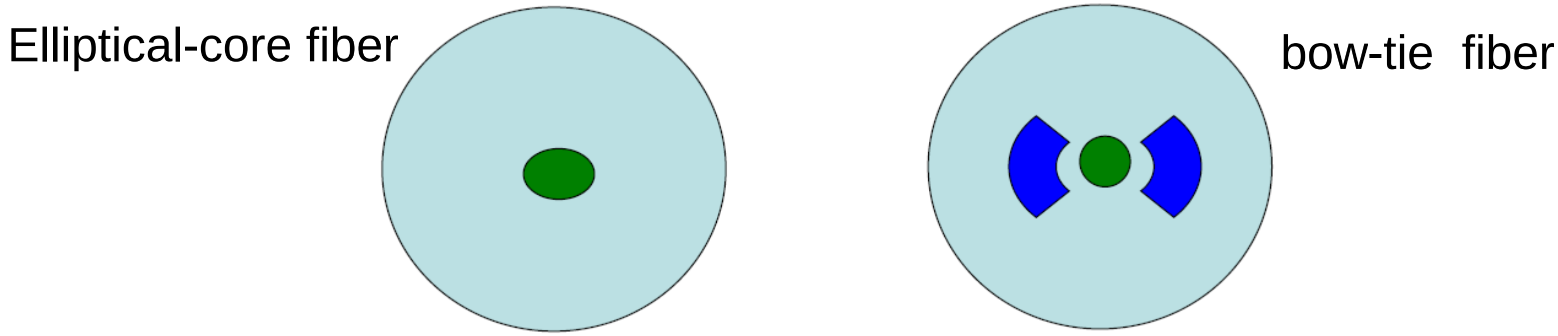
Polarizing effects of conventional / polarization-preserving fibers

OF type	Input	Output
conventional	Unpolarized input	Unknown output (random coupling between all the polarizations present)
	Polarized input	Unknown output
polarization-preserving	Unpolarized input	Unknown output
	Polarized input	Polarized output

- The fiber birefringence is enhanced in single-mode polarization-preserving (polarization-maintaining) fibers, which are designed to maintain the polarization of the launched wave.
- Polarization is preserved because the two possible waves have significantly different propagation characteristics. This keeps them from exchanging energy as they propagate through the fiber.

Polarization-preserving fibers

- Polarization-preserving fibers are constructed by designing asymmetries into the fiber. Examples include fibers with elliptical cores (which cause waves polarized along the major and minor axes of the ellipse to have different effective refractive indices) and fibers that contain nonsymmetrical stress-producing parts.



The shaded region in the bow-tie fiber is highly doped with a material such as boron. Because the thermal expansion of this doped region is so different from that of the pure silica cladding, a nonsymmetrical stress is exerted on the core. This produces a large stress-induced birefringence, which in turn decouples the two orthogonal modes of the singlemode fiber.

SMF – Commercial fiber

Characteristics of several commercial fibers

Fiber Type and Trade Name	A_{eff} (μm^2)	λ_{ZD} (nm)	D (C band) [ps/(km-nm)]	Slope S [ps/(km-nm^2)]
Corning SMF-28	80	1302–1322	16 to 19	0.090
OFS AllWave	80	1300–1322	17 to 20	0.088
Draka ColorLock	80	1300–1320	16 to 19	0.090
Corning Vascade	100	1300–1310	18 to 20	0.060
OFS TrueWave-RS	50	1470–1490	2.6 to 6	0.050
Corning LEAF	72	1490–1500	2.0 to 6	0.060
Draka TeraLight	65	1430–1440	5.5 to 10	0.052

Chirped Gaussian pulses

A pulse is said to be chirped if its carrier frequency changes linearly with time $\frac{\mathrm{d}\omega}{\mathrm{d}t} = \mathrm{const} = \frac{C}{T_0^2}$

continuous wave (E): $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\hat{\mathbf{x}} F(x,y) A(z,t) \cdot \exp(j\beta_0 z - j\omega_0 t)\}$

- Gaussian pulse - field $A(0,t) = A_0 \exp\left[-1/2 \cdot (1 + jC) \cdot (t/T_0)^2\right]$

- intensity $|A(0,t)|^2 = A_0^2 \exp\left[-1 + jC \cdot (t/T_0)^2\right]$

– A_0 is the peak amplitude

– C is the chirp parameter

– T_0 is the time at 1/e of maximum power

- after Fourier Transform

$$\tilde{A}(0,\omega) = A_0 \sqrt{\frac{2\pi T_0^2}{1 + jC}} \cdot \exp\left[\frac{-\omega^2 T_0^2}{2(1 + jC)}\right]$$

Chirp frequency - Time-bandwidth product

- full width at half of 1/e · maximum (FWHM) $T_{FWHM} = 2\sqrt{\ln(2)} \cdot T_0 \approx 1.665 T_0$

$$\left|\tilde{A}(0,\omega)\right|^2 = const \cdot \exp\left[\frac{-\omega^2 T_0^2}{(1+jC)}\right] = const \cdot \exp\left[\frac{-\omega^2 T_0^2}{(1+C^2)}\right] \cdot \exp\left[\frac{-j\omega^2 T_0^2 C}{(1+C^2)}\right]$$

$$\ln\left(\exp\left[\frac{-\omega^2 T_0^2}{(1+C^2)}\right]\right) = \ln\left(\frac{1}{e}\right) \Rightarrow -\omega^2 T_0^2 = -(1+C^2) \Rightarrow \Delta\omega = \frac{\sqrt{1+C^2}}{T_0}$$

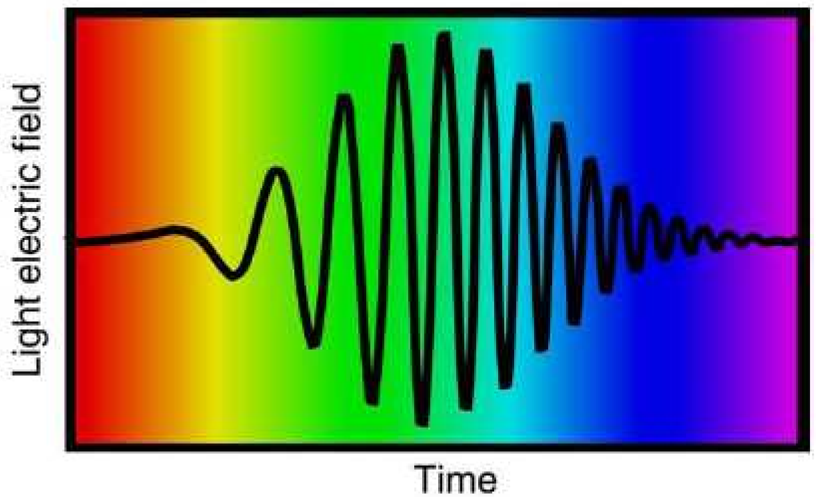
Spectral width is enhanced by a factor of $\sqrt{1+C^2}$

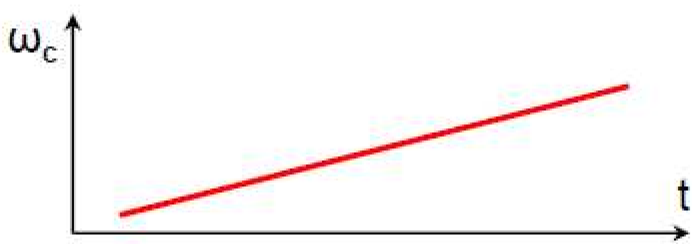
For C=0, ($\Delta\omega \cdot T_0$=1) the pulse has the narrowest spectrum and is called ***Transform-Limited*** (pulse with lowest duration that is possible for a given optical spectrum of the pulse ↔ for given pulse duration *Transform-Limited* pulse are that with the minimum possible spectral width).

$$\Delta f_{FWHM} \cdot T_{FWHM} = 2 \cdot \ln 2/\pi \sqrt{1+C^2} \approx 0.44\sqrt{1+C^2}$$

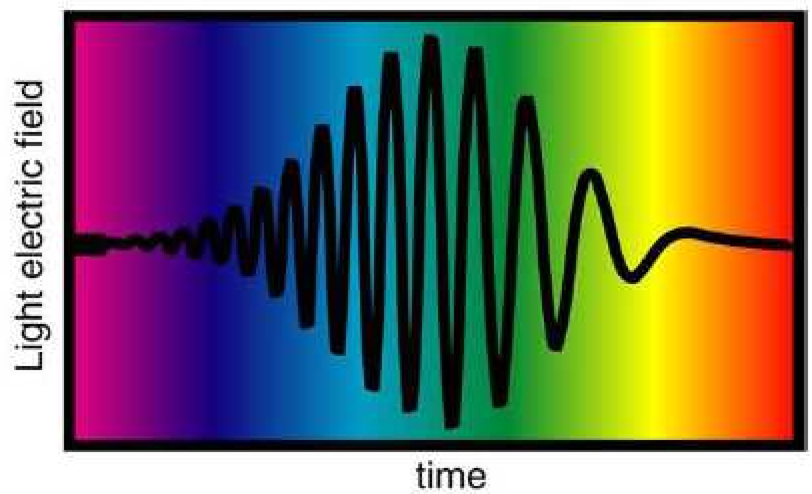
A linearly chirped pulse

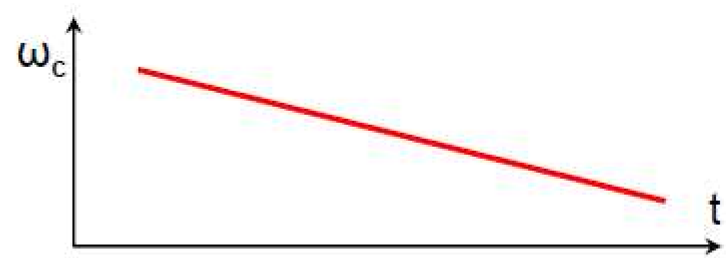
Frequency increases with time



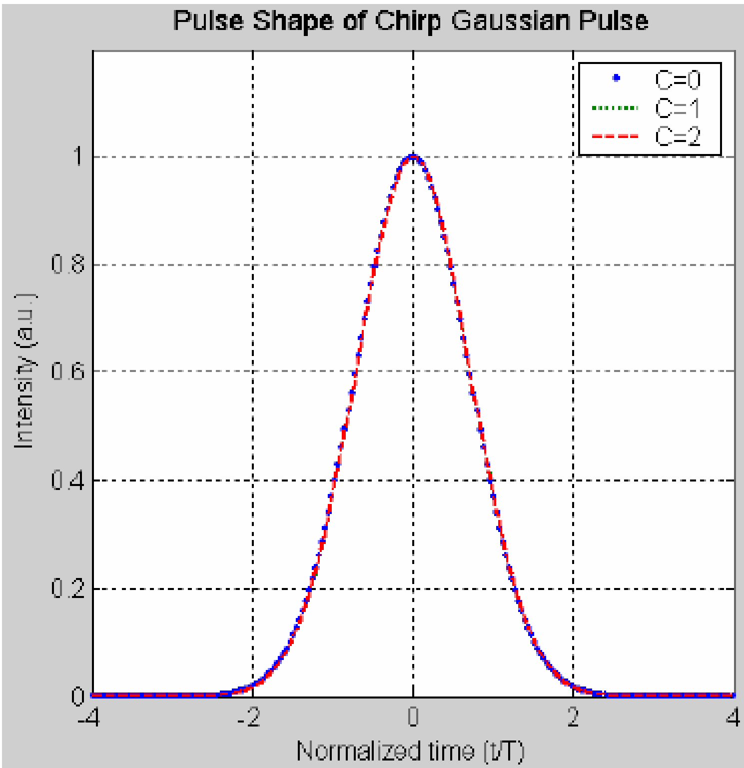


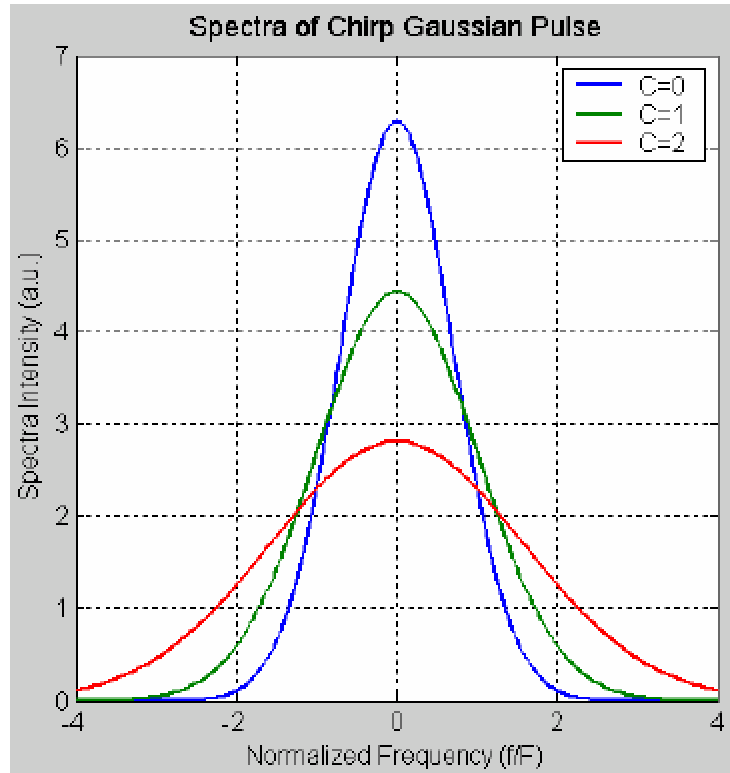
Frequency decreases with time





Chirped Gaussian Pulses Width and Spectra





Intensity pulse shape and spectra for Chirp Gaussian Pulse.

Chirped Gaussian pulses

We introduce $\xi = z/L_D$ where the dispersion length $L_D = T_0^2 / |\beta_2|$

In the time domain the dispersed pulse is

$$A(\xi,t) = \frac{A_0}{\sqrt{b_f}} \exp\left[-\frac{(1+jC_1)\cdot t^2}{2T_0^2 b_f^2} + \frac{j}{2}\arctan\left(\frac{\xi}{1+C\xi}\right)\right]$$

$$b_f(\xi) = \sqrt{(1+sC\xi)^2 + \xi^2}$$
$$C_1(\xi) = C + s(1+C^2)\xi$$
$$s = \mathrm{sign}(\beta_2)$$

The output width (1/e-intensity point) broadens as

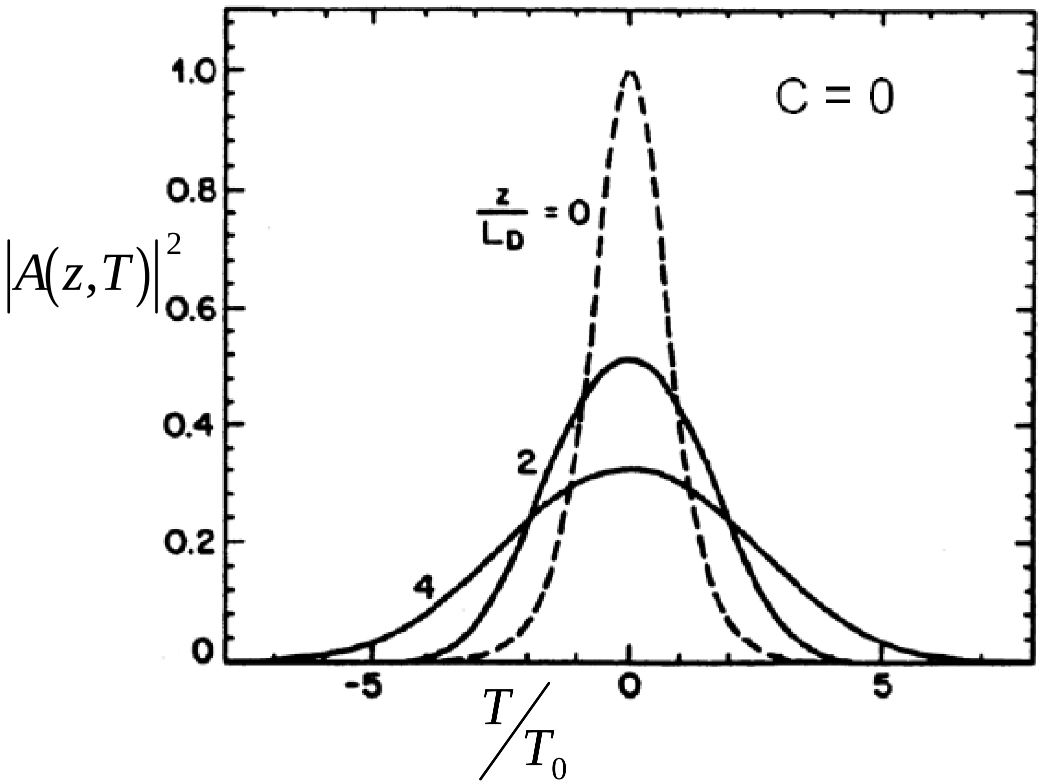
$$b_f(z) = \frac{T_1(z)}{T_0} = \sqrt{\left(1+\frac{C\beta_2 z}{T_0^2}\right)^2 + \left(\frac{\beta_2 z}{T_0^2}\right)^2}$$

A Gaussian pulse remains Gaussian during propagation

The chirp, $C_1(\xi)$, evolves as the pulse propagates

If ($C\cdot\beta_2$) is negative, the pulse will initially be compressed

Broadening of chirp-free Gaussian pulses

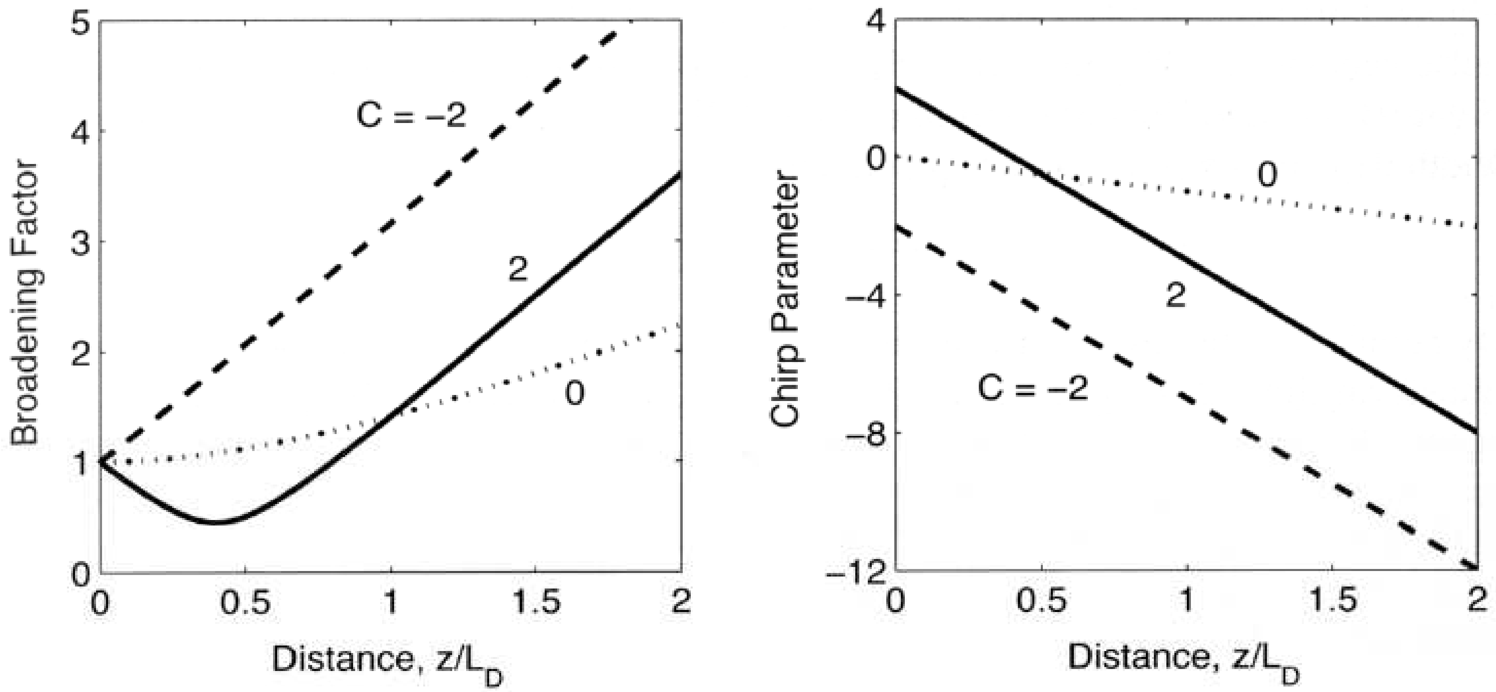


$$b_f(z) = \sqrt{1 + \left(\frac{z}{L_D}\right)^2} = \sqrt{1 + \left(\frac{|\beta_2| z}{T_0^2}\right)^2}$$

Short pulses broaden more quickly than longer pulses

(Compare with diffraction of beams)

Broadening of linearly chirped Gaussian pulses



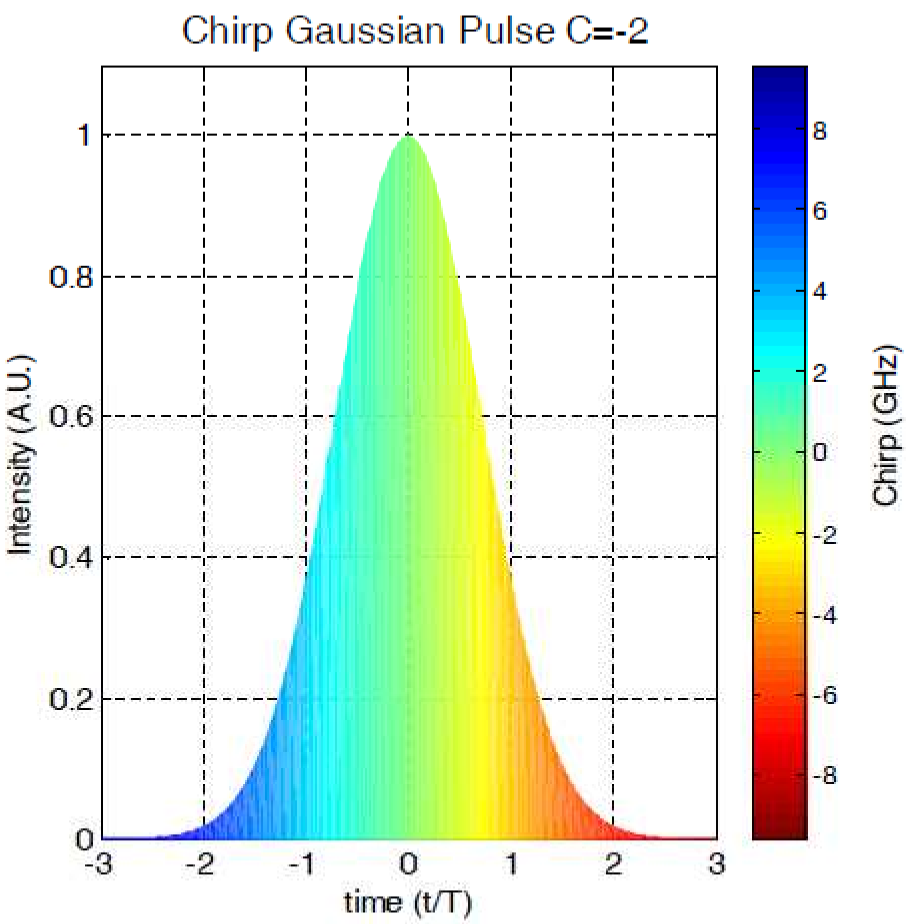
For $(C\,\beta_2) < 0$, pulses initially compress and reaches a minimum at

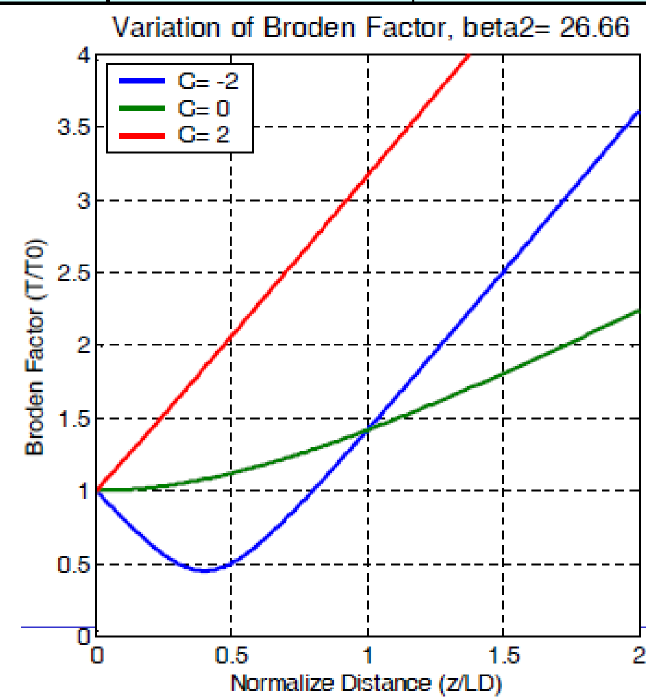
$z = |C|/(1+C^2)L_D$ at which $C_1 = 0$ and $$T_1^{\min} = \frac{T_0}{\sqrt{1+C^2}} = \frac{1}{\Delta\omega_0}$$

Chirped pulses eventually broaden more quickly than unchirped pulses

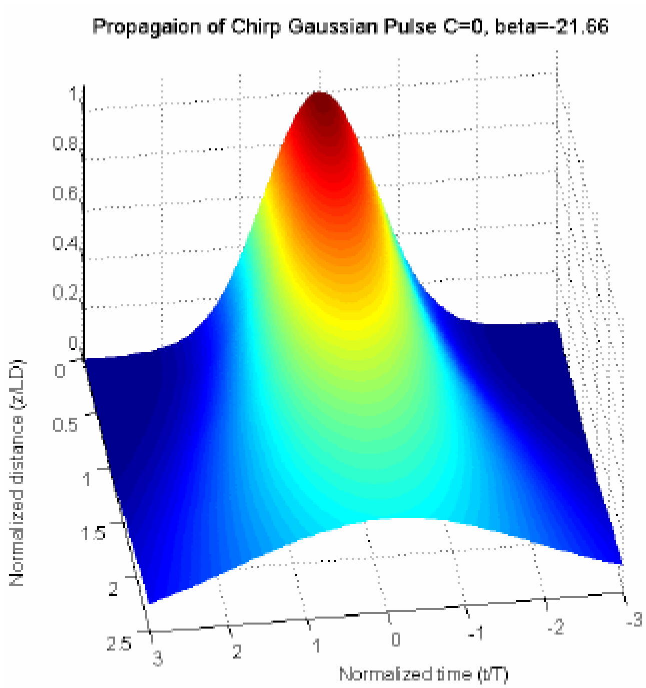
Broadening of linearly chirped Gaussian pulses

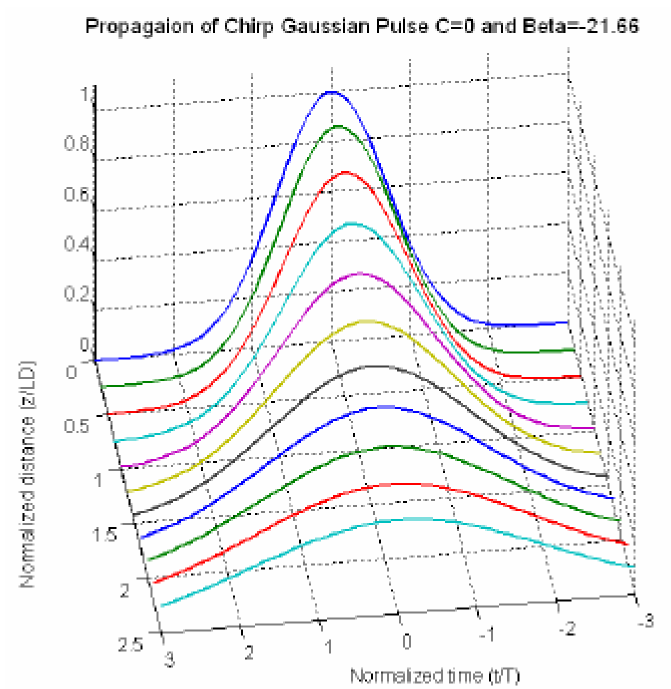
	$\beta_2 > 0$:Normal		$\beta_2 < 0$: Anomalous	
Initial or C=0	Blue: Slower	Red: Faster	Blue: Faster	Red: Slower
C > 0	Leading: Red			
	Continue Widen		Narrowing first then widen	
C < 0	Leading: Blue			
	Narrowing first then widen		Continue Widen	





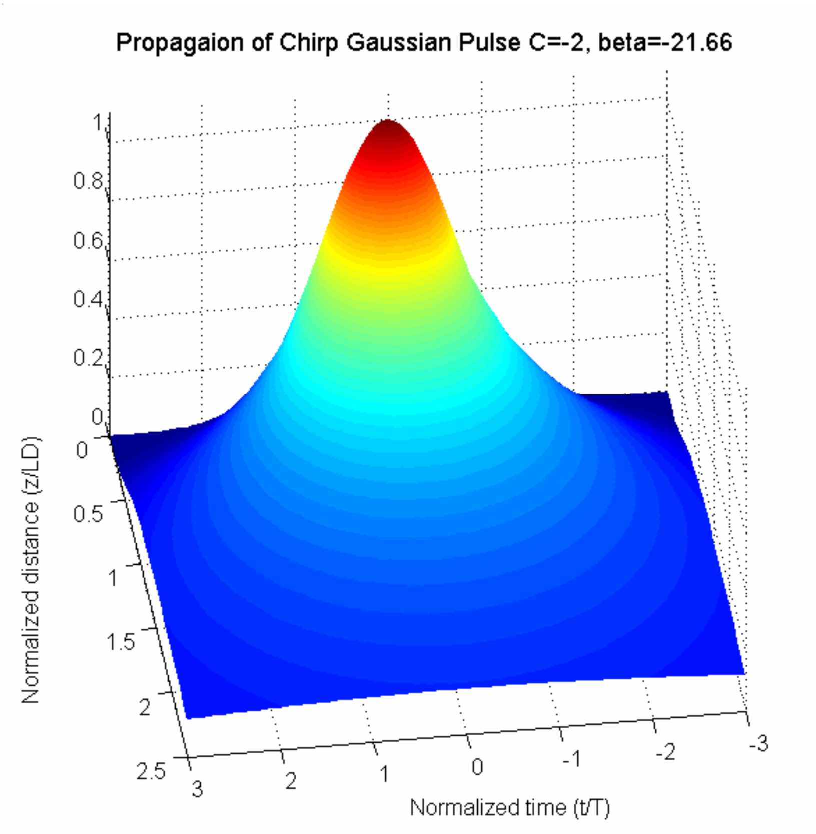
Chirped Gaussian Pulses C=0, *β*=-21.66

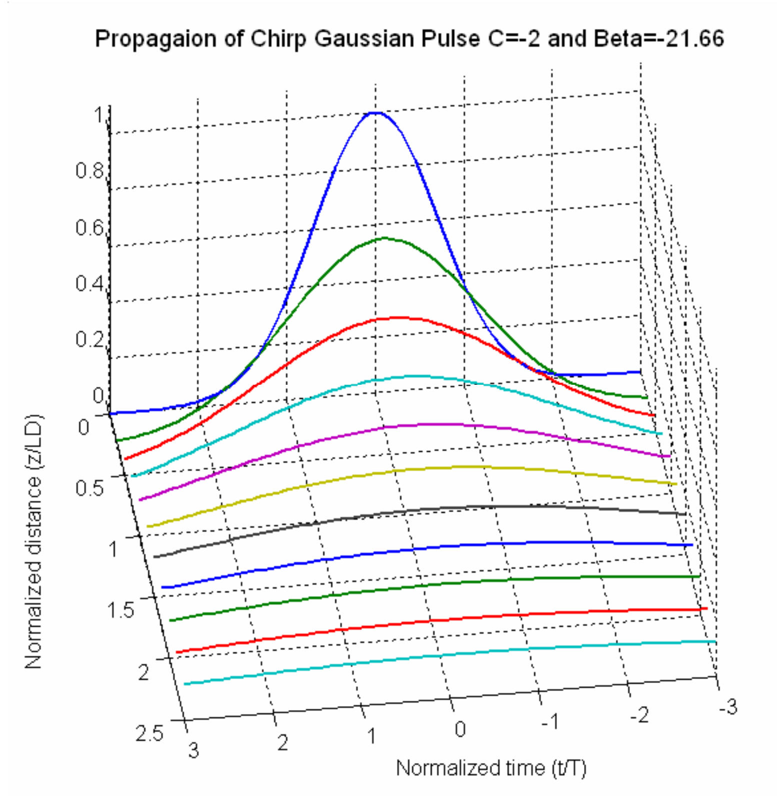




Propagation of Chirp Gaussian pulse

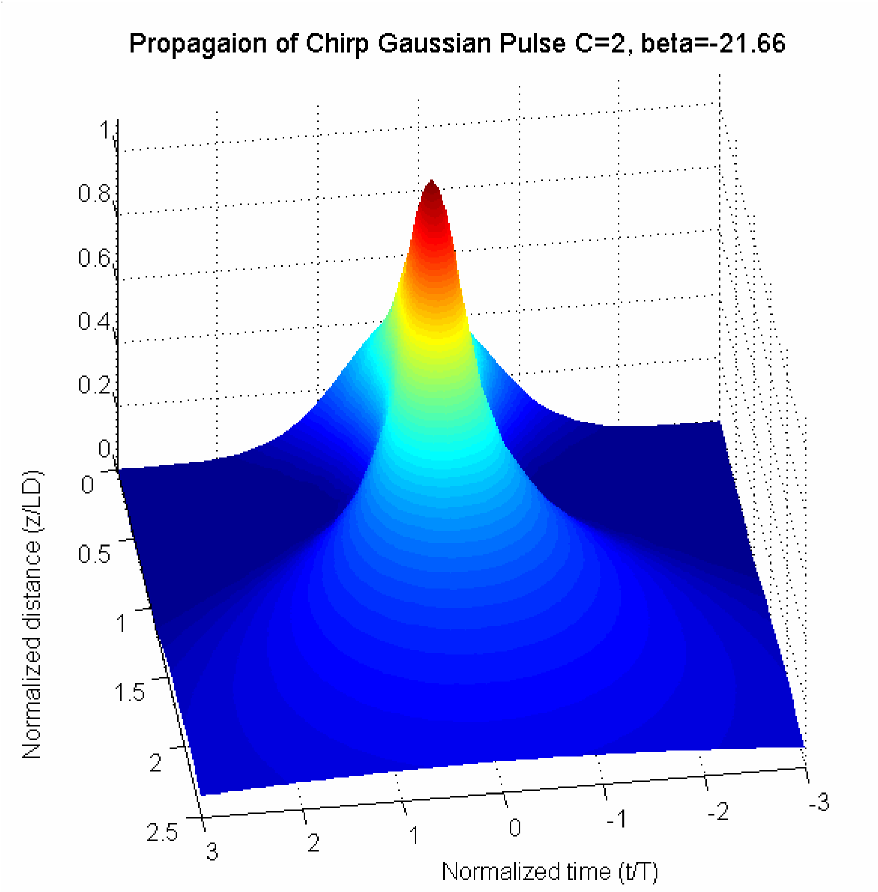
Chirped Gaussian Pulses C=0, β=-21.66

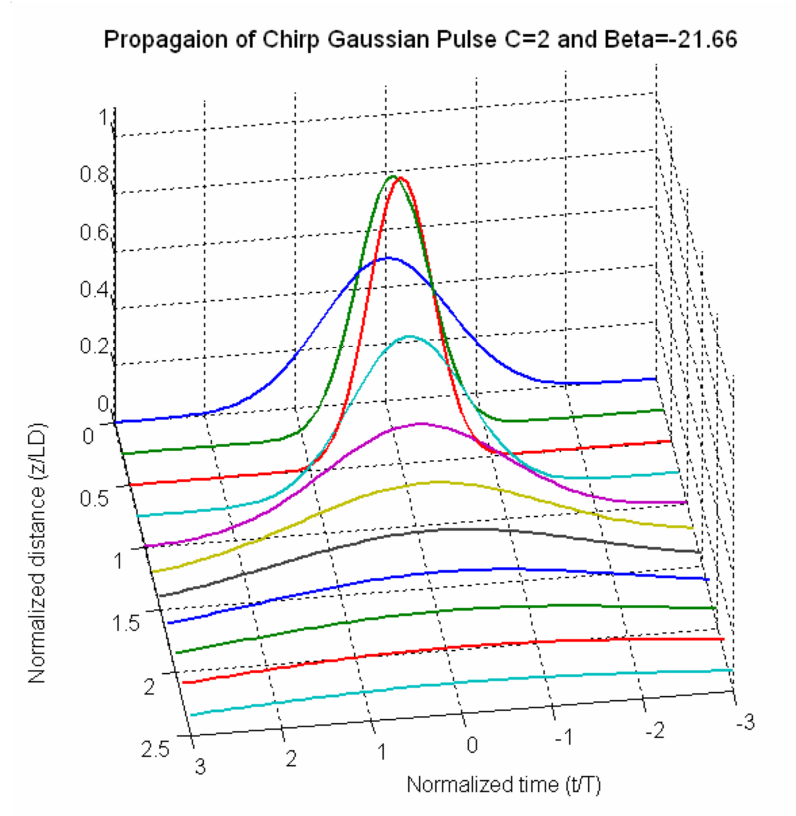




Propagation of Chirp Gaussian pulse

Chirped Gaussian Pulses C=0, β=-21.66





Propagation of Chirp Gaussian pulse

Non-Gaussian pulses

- Only Gaussian pulses remain Gaussian upon propagation;
- Not even Gaussian pulses remain Gaussian if β_3 cannot be ignored;
- In these cases, the RMS-pulse width can be used;

$$\sigma_p^2 = \langle t^2 \rangle - \langle t \rangle^2 \qquad ; \qquad \langle t^m \rangle = \frac{\int_{-\infty}^{+\infty} t^m |A(z,t)|^2 \, \mathrm{d}t}{\int_{-\infty}^{+\infty} |A(z,t)|^2 \, \mathrm{d}t}$$

- σ_p can be calculated when the initial pulse is known

Example 1: An unchirped rectangular pulse will, in presence of only β_2, broaden as

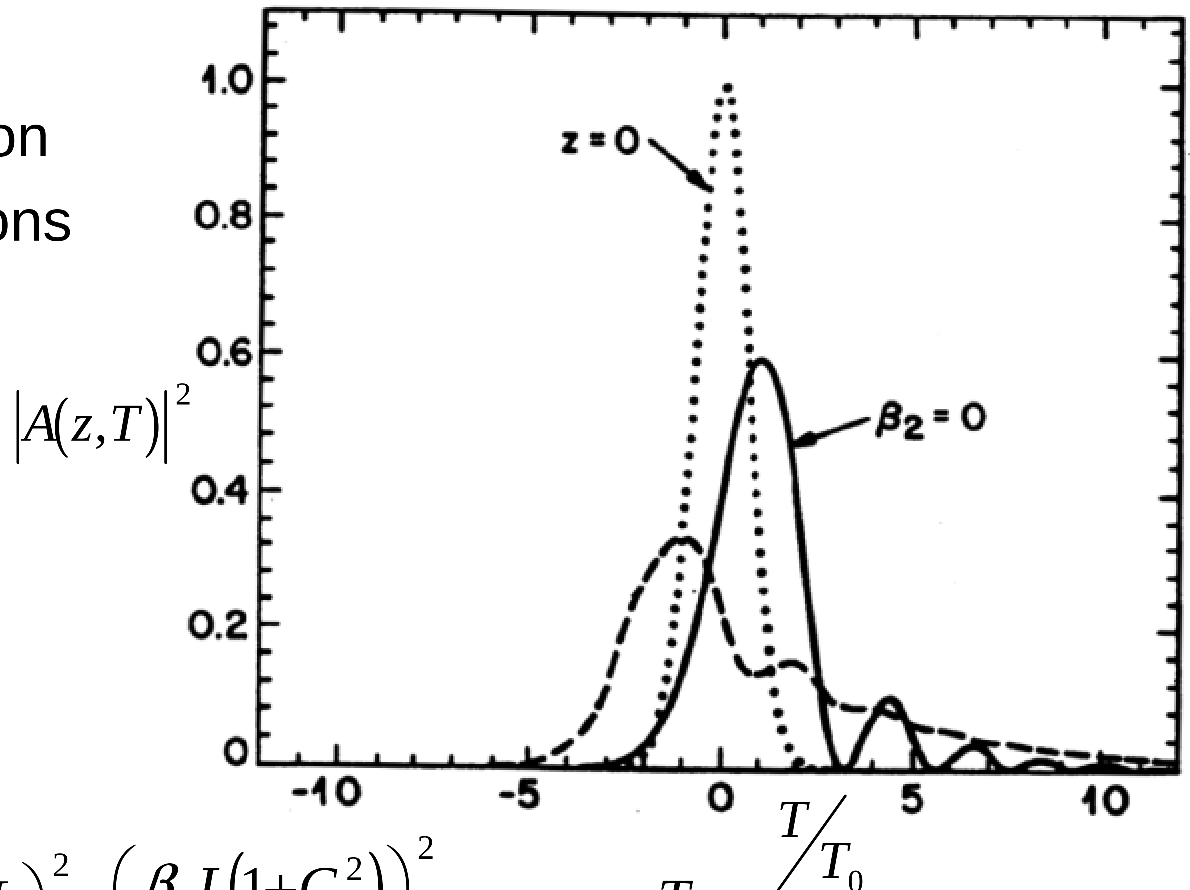
$$\sigma_p^2 = \sigma_0^2 \left[1 + \frac{3}{2} \left(\frac{L}{L_D} \right)^2 \right]$$

Example 2: An unchirped Gaussian pulse will, in presence of only β_2, broaden as

$$\sigma_p^2 = \sigma_0^2 \left[1 + \left(\frac{L}{L_D} \right)^2 \right]$$

Chirped Gaussian pulses in the presence of β_3

Higher order dispersion gives rise to oscillations and pulse shape changes)



$$\frac{\sigma^2}{\sigma_0^2} = \left(1 + \frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_3 L\left(1+C^2\right)}{4\sqrt{2}\sigma_0^3}\right)^2 \quad ; \ \sigma_0 = \frac{T_0}{\sqrt{2}}$$

Chirped Gaussian pulses in the presence of β$_3$

- Using a light source with a broad spectrum leads to strong dispersive broadening of the signal pulses.

In practice, this only needs to be considered when the source spectral width approaches the symbol rate

For a Gaussian-shaped source spectrum with RMS-width σ_ω and with Gaussian pulses, we have

$$\frac{\sigma^2}{\sigma_0^2} = \left(1 + \frac{C\beta_2 L}{2\sigma_0^2}\right)^2 + \left(1 + V_\omega^2\right)\left(\frac{\beta_2 L}{2\sigma_0^2}\right)^2 + \left(1 + C^2 + V_\omega^2\right)^2 \cdot \left(\frac{\beta_3 L}{4\sqrt{2}\sigma_0^3}\right)^2$$

where $V_\omega = 2\sigma_\omega\sigma_0$

$V_\omega << 1$ when the source spectral width << the signal spectral width

Limitations on bit rate, incoherent source

If $V_\omega >> 1$ (the light source spectral width is much larger than the signal spectral width, as for LED) we obtain approximately

$$\sigma^2 \simeq \sigma_0^2 + \left(\beta_2 L \sigma_\omega\right)^2 \equiv \sigma_0^2 + \left(D L \sigma_\lambda\right)^2$$

A common criteria for the bit rate is that $\sigma \leq T_B/4 = 1/(4B)$

For the Gaussian pulse, this means that 95% of the pulse energy remains within the bit slot

In the limit of large broadening $4B \cdot |D| L \sigma_\lambda \leq 1$

σ_λ is the source RMS width in wavelength units.

Exp: D= 17 ps/(km nm), σ_λ =15 nm ⇒ $(BL)_{max} \approx 1$ (Gbit/s) km

Limitations on bit rate, incoherent source

In the case of operation at $\lambda = \lambda_{ZD}$, $\beta_2 = 0$ we have

$$\sigma^2 = \sigma_0^2 + 1/2 \cdot \left(\beta_3 L \sigma_\omega^2\right)^2 \equiv \sigma_0^2 + \left(S L \sigma_\lambda^2\right)^2$$

With the same condition on the pulse broadening, we obtain

$$8B \cdot |S| L \sigma_\lambda^2 \leq 1$$

The dispersion slope, S, will determine the bit rate-distance product

Exp: $D = 0$, S= 0.08 ps/(km ·nm^2), σ_λ =15 nm ⇒ (BL)max ≈ 20 (Gbit/s) km

Limitations on bit rate, coherent source

For most lasers $V_\omega \ll 1$ and can be neglected and the criteria become , neglecting β_3:

$$\sigma^2 = \sigma_0^2 + \left(\beta_2 L / (2\sigma_\omega)\right)^2 \equiv \sigma_0^2 + \sigma_D^2$$

The output pulse width is minimized for a certain input pulse width giving

$$4B \cdot \sqrt{|\beta_2| L} \le 1$$

Example: β_2 = 20 ps^2/km → (B^2L)max ≈ 3000 (Gbit/s)^2 km

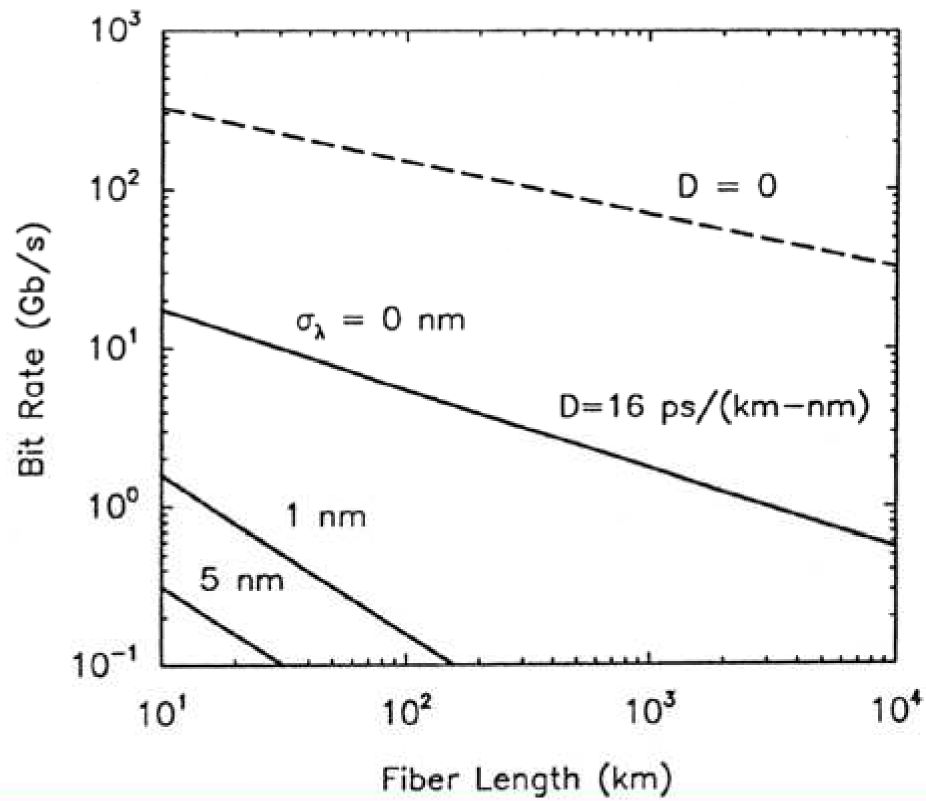
500 km @ 2.5 Gbit/s, 30 km @ 10 Gbit/s

If $\beta_2 = 0$ (close to λ_0): $\sigma^2 = \sigma_0^2 + \left(\beta_3 L / (4\sigma_0)\right)^2 / 2 \equiv \sigma_0^2 + \sigma_D^2$

For an optimal input pulse width, we get

$$\sigma^2 = \sigma_0^2 + \left(B \cdot |\beta_3| L\right)^{1/3} \le 0.324$$

Limitations on bit rate, summary



A coherent source improves the bit rate-distance product

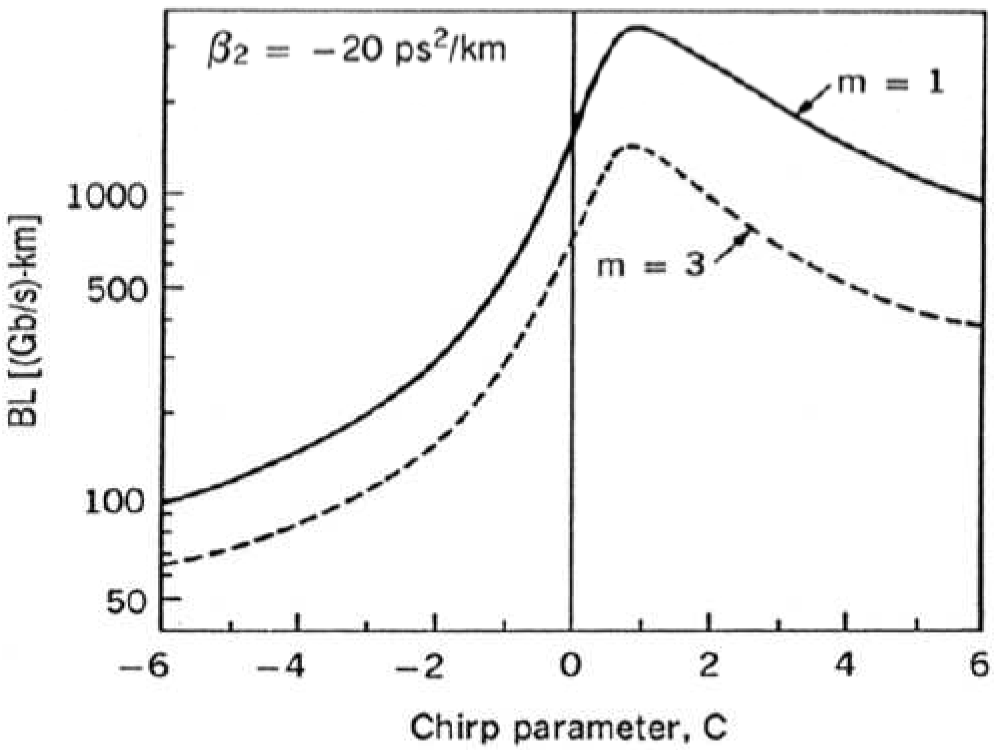
Operation near the zero-dispersion wavelength also is beneficial…

…but may lead to problems with nonlinear signal distortion

Super-Gaussian pulses

- Super-Gaussian pulses are flat-top and can be used to model NRZ…
  - …but more accurate modeling is preferred
  - If m = 1 ⇒ We recover the Gaussian
  - If m > 1 ⇒ The shape is more rectangular

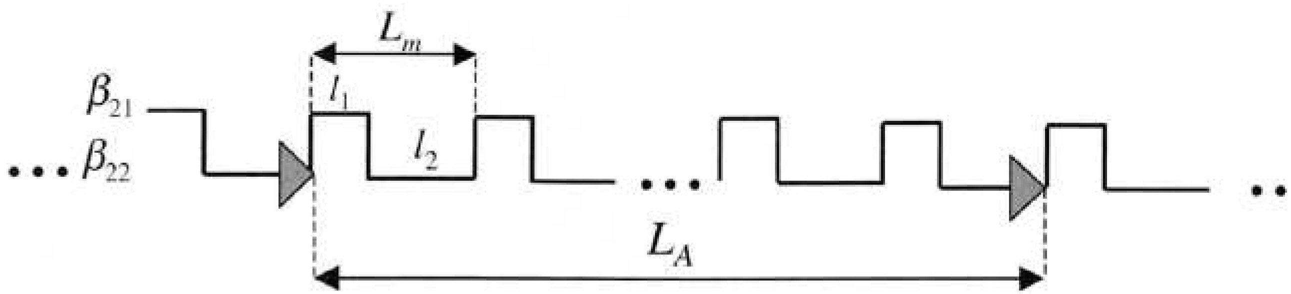
$$A(0,t) = A_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^{2m} \right]$$



Numerical simulation shows:

- Super-Gaussian has:
  - Smaller bit rate-distance product…
  - …due to more problems with dispersion…
  - …due to the sharper edges
- A small chirp can be beneficial:
  - $C \approx 1$ is optimal

Dispersion compensation



- Dispersion is a key limiting factor for an optical transmission system
- Several ways to compensate for the dispersion exist
  - More about this in a later lecture...
- One way is to periodically insert fiber with opposite sign of *D*
  - This is called ***dispersion-compensating fiber*** (DCF)
  - Figure shows a system with both SMF and DCF
  - The GVD parameters are β_{21} and β_{22}
- Group-velocity dispersion is perfectly compensated when

  $\beta_{21} l_1 + \beta_{22} l_2 = 0$, which is equivalent to $D_1 l_1 + D_2 l_2 = 0$
- GVD and PMD can also be compensated in digital signal processing (DSP)

Example of practical fiber

Multimode Fiber Transmission Distances

- The possible transmission distances when using fibers with different core sizes and bandwidths for Ethernet, Fibre Channel, and SONET/SDH applications

Table 3.3 *Transmission distances in meters in multimode fibers using an 850-nm VCSEL*

		50-μm core		*62.5-μm core*	
Application	Data rate (Gb/s)	500 MHz.km	2000 MHz.km	160 MHz.km	200 MHz.km
Ethernet	1	550	860	220	275
	10	82	300	26	33
Fibre Channel	1	500	860	250	300
	2	300	500	120	150
	10	82	300	26	33
SONET/SDH	10	85	300	25	33

Examples of Specialty Fibers

Table 3.4 *Examples of specialty fibers and their applications*

Specialty fiber type	*Application*
Erbium-doped fiber	Gain medium for optical fiber amplifiers
Photosensitive fibers	Fabrication of fiber Bragg gratings
Bend-insensitive fibers	Tightly looped connections in device packages
Termination fiber	Termination of open optical fiber ends
Polarization-preserving fibers	Pump lasers, polarization-sensitive devices, sensors
High-index fibers	Fused couplers, short-λ sources, DWDM devices
Photonic crystal fibers	Switches; dispersion compensation

ITU-T Recommendations for Fibers (1)

Table 3.2 *Recommendations for fibers used in telecom, access, and enterprise networks*

ITU-T rec. no.	*Title and description*
G.651.1 (Edition 1, July 2007); Addendum (Dec. 2008)	Title: *Characteristics of a 50/125 μm multimode graded index optical fiber cable for the optical access network*
	Description: Gives the requirements of a silica 50/125 μm multimode graded index optical fiber cable for use in the 850-nm or 1300-nm regions, either individually or simultaneously
G.652 (Edition 8, Nov. 2009)	Title: *Characteristics of a Single-Mode Optical Fiber and Cable*
	Description: Discusses single-mode fiber optimized for O-band (1310-nm) use, but which also can be used in the 1550-nm region
G.653 (Edition 6, Dec. 2006)	Title: *Characteristics of a Dispersion-Shifted Single-Mode Optical Fiber and Cable*
	Description: Discusses single-mode optical fiber with the zero-dispersion wavelength shifted into the 1550 nm region. Describes chromatic dispersion for the 1460-to-1625-nm range for CWDM applications
G.654 (Edition 7, Dec. 2006)	Title: *Characteristics of a Cut-Off Shifted Single-Mode Optical Fiber and Cable*
	Description: Undersea applications; discusses single-mode optical fiber with a zero-dispersion wavelength around 1300 nm and with cutoff wavelength shifted to around 1550 nm

ITU-T Recommendations for Fibers (2)

ITU-T rec. no.	*Title and description*
G.655 (Edition 5, Nov. 2009)	Title: *Characteristics of a Non-Zero Dispersion-Shifted Single-Mode Optical Fiber and Cable* Description: For applications in long-haul links; describes single-mode optical fiber with chromatic dispersion greater than zero throughout the 1530-to-1565-nm wavelength range
G.656 (Edition 2, Dec. 2006)	Title: *Characteristics of a Fiber and Cable with Non-Zero Dispersion for Wideband Optical Transport* Description: Low chromatic dispersion fiber for expanded WDM applications; can be used for both CWDM and DWDM systems throughout the wavelength region between 1460 and 1625 nm
G.657 (Edition 2, Nov. 2009)	Title: *Characteristics of a bending loss insensitive single-mode optical fiber and cable for the access network* Description: Addresses use of single-mode fiber for broadband access networks; includes issues such as sensitivity to tight bending conditions for in-building use

Bibliography

- G. P. Agrawal – Fiber Optic Communication Systems, 4th Ed, 2010

- R. Ramaswam, et. at. - Optical Networks. A Practical perspective, 3rd Ed, 2010

- J. M. Senior -Optical Fiber Communications Principles and Practice, 3rd, Ed 2009