Laboratory 2
Measurement of the nonlinearity distortions

Purpose: Measurement of the distortions for different types of signals. Measurement of the distortions for an elementary amplification stage.

Summary of theory

A stable circuit, having time invariant parameters, is linear if the output signal may be written as a linear function (1st order polynomial) of the input signal, when the signals are sinusoids. In other terms, the output signal has the same spectral components as the input signal (all or part of them). One considers that a circuit is nonlinear if its output signal has spectral components which cannot be found in the input signal. Therefore, if at the input of a circuit one applies the signal:

$$x(t)=A\cdot\cos(2\pi f_1 t)+B\cdot\cos(2\pi f_2 t) \tag{1}$$

at the output, the signal $y(t)$ may have spectral components on the frequencies:

$$pf_1+qf_2>0 \quad \text{where } p, q \in \mathbf{Z}^* \tag{2}$$

The circuit is linear if its output signal has spectral components only on f_1 and/or f_2 ({p=1, q=1}, {p=0, q=1}, {p=1, q=0}). For other values of p and q the circuit is nonlinear. Spectral components introduced by the nonlinear circuit are named *nonlinearity distortions.*

Consequently, for nonlinear circuits the spectrum of the output signal has the following components:

- harmonics of frequencies f_1 and f_2 (frequencies equal to $p\cdot f_1$ and $q\cdot f_2$, with $p, q \in \mathbf{Z}^* \setminus \{1\}$).
- intermodulation products (frequencies equal to $p\cdot f1+q\cdot f2$, with $p, q \in \mathbf{Z}^*$).

The *distortion meter* is used to measure the equivalent RMS value of all spectral components (except the fundamental component) compared to the RMS value of the entire signal. Also, using *the distortion meter* one can evaluate the distortion degree introduced, in practice, by a linear circuit. To that effect, one measures the distortion degree of a sinusoidal test signal (that has a single spectral component - the fundamental component) applied at the circuit input and the distortion degree of the output signal.

Usually, the nonlinearity of a circuit is due to the nonlinear dependence between current and voltage, specific to active devices.

The distortion meter

For a sinusoidal signal $u(t)$ without DC bias, with the period $T_0=2\pi/\omega_0$ the distortion factor with respect to the effective value THD (Total Harmonic Distortion) is defined as follows:

$$THD=\frac{U_{ef,harmonics}}{U_{ef,signal}}=\frac{\sqrt{\sum_{k=2}^{\infty}\frac{U_k^2}{2}}}{\sqrt{\sum_{k=1}^{\infty}\frac{U_k^2}{2}}} \tag{3}$$

where U_k represents the amplitude of the kth harmonic (k=1 for the fundamental component).

As one can see, the distortion degree points out the degree of resemblance between the measured signal and the sinusoidal signal. The distortion factor of an ideal sinusoidal signal has a distortion factor equal to 0% (as it has no harmonics).

A periodic signal *u(t)* can be decomposed in Harmonic Fourier Series:

$$u(t)=\sum_{k=1}^{\infty}U_k\cos(k\omega_0 t+\phi_k)=\sum_{k=1}^{\infty}U_k\cos(2\pi k f_0 t+\phi_k) \tag{4}$$

where U_1 is the amplitude of the fundamental component, and U_k (k = 2, 3, 4, …) are the amplitudes of the harmonics. Using Parceval's theorem, the distortion factor can be rewritten as follows:

$$THD=\frac{\sqrt{\sum_{k=2}^{\infty}U_k^2}}{\sqrt{\sum_{k=1}^{\infty}U_k^2}}=\sqrt{\frac{U_{ef,signal}^2-\frac{1}{2}U_1^2}{U_{ef,signal}^2}} \tag{5}$$

Given the way the distortion factor is defined, a schematic diagram for measuring *THD* will look like the one in Fig. 1.

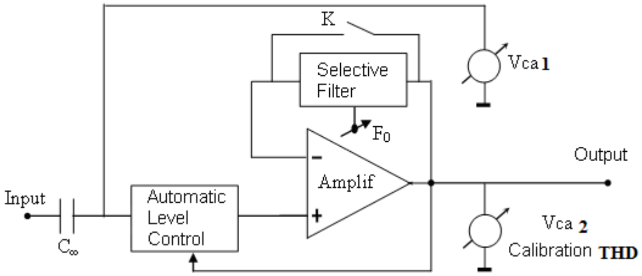


Fig. 1: Distortion meter – overall schematic

One can notice the *automatic level control* block that has the role of automatically adjusting the RMS value of the signal, which is the denominator of the equations (3) and (5), to 1 V (0 dB). During the stage of this adjustment, the K switch is automatically *closed*, thus obtaining a *repeater (buffer)* configuration for the operational amplifier circuit. Therefore, one obtains a signal that is identic with the input signal, having, though, U_{RMS} =1V, the *normalized signal* that is observed at the X output (with respect to GND) of the distortion meter.

In the second stage of the measurement, the switch K opens, also automatically, and the negative feedback operational amplifier becomes an active rejection filter (a stop-band filter with a very small stop-band called notch filter). This filter has the purpose of eliminating the fundamental frequency of the normalized signal, with U_{RMS} (signal)=1V. The resulting signal is available at the Y output (with respect to GND) of the distortion meter. The AC voltmeter, Vca2, measures the effective value of this signal (the normalized input signal without its fundamental component) which represents the value of the numerator of equations (3) and (5), hence, the *distortion factor*.

The rejection filter is adjustable, its frequency can be adjusted through the RANGE mode adjustments (SPOT/RANGE button not-pressed) and the ***Tuning Freq knob***, for GAD-201G distortion meter in this laboratory.

The C_∞ capacitor is used to eliminate the DC bias from the input signal, as it would disturb the measurement of the distortion factor because it is not eliminated by the notch filter, and it would erroneously add to the power of the distortion signal.
In addition, the AC voltmeter Vca 1 allows the measurement of the effective value of the input signal, before being normalized.

The previous method implies the assumption that the SNR (signal-to-noise ratio) has values high enough so that the harmonics have magnitudes much bigger than the noise in the frequency bandwidth of interest. One considers that the noise is the signal composed of all spectral components that are not on the frequencies obtained through equation (2). In this case, the equivalent effective voltage of the noise is negligible and is not considered in the formula.

If the noise is not negligible (especially when the measurements are performed for low signal operation), the noise must be considered and THD becomes THD+N (*Total Harmonic Distortion plus Noise*):

$$THD+N=\frac{U_{ef,\,harmonics+noise}}{U_{ef,signal}}=\frac{\sqrt{\sum_{k=2}^{\infty}\frac{U_k^{\,2}}{2}+U_{ef,noise}^2}}{U_{ef,signal}} \qquad (6)$$

Usually, one uses also *Signal to Noise and Distortion* ratio, SINAD, which is the inverse of *THD+N*.

$$SINAD=\frac{U_{ef,signal}}{\sqrt{\sum_{k=2}^{\infty}U_k^{\,2}+U_{ef,noise}^{\,2}}} \qquad (7)$$

Generally, THD is expressed in percent or dB; for the latter:

$$THD_R+N\ [dB]= -\ SINAD\ [dB] \qquad (8)$$

Remarks:

1. The level control and the switch K are automatically operated in GAD-201G distortion meter, thus human intervention is not necessary. Obviously, for other types of distortion meters that do not have automatic adjustments, the two steps must be followed separately through human intervention.

2. The distortion factor, as it was defined, *is meaningful only for sinusoidal periodic signals*. For other signal shapes, it represents the ratio between the RMS voltage of the signal and the RMS voltage of the signal without the fundamental component. Thus, for those signals, the distortion factor represents their deviation from the sinusoidal signal having the same period as the measured signal.

3. The values of THD for some signals are well-known:

Rectangular signal	$THD=\sqrt{\frac{\pi^2}{8}-1}$	Sawtooth signal	$THD=\sqrt{\frac{\pi^2}{6}-1}$
Symmetric Triangular signal	$THD=\sqrt{\frac{\pi^4}{96}-1}$	Rectangular pulses with η duty cycle	$THD=\sqrt{\frac{\mu(1-\mu)\pi^2}{2\sin^2(\pi\mu)}-1}$

Operating modes of the Distortion Meter GAD-201G

1. Distortions measurement

Characteristics:

⇒ The device is capable of automatic calibration - automatic level control of the input signal through bringing the needle of the voltmeter that points the distortion factor (*THD*) to 100%.
⇒ The device automatically tunes the frequency of the rejection filter to the fundamental frequency of the input signal. This happens when the two frequencies are close enough. The double of the maximal distance between the two frequencies that allows automatic tuning is called the distortion meter ***capture bandwidth.***

Operating modes:

- C***ontinuous mode*** operation (knob SPOT/RANGE not-pressed) – this operating mode allows measuring distortions at any frequency in the measurement range (20Hz – 20 kHz). Once the signal is applied, the user must adjust the frequency of the notch filter near the frequency of the signal. Then, the device finely and automatically adjusts the rejection frequency until it matches the frequency of the signal. To adjust the frequency of the notch filter, turn the knob ***Tuning Freq*** in the sense indicated by the red LEDs ***High*** or ***Low*** next to the indication. When the input frequency is within the limits of automatic adjustment (the *capture bandwidth*), neither LED is lit and the device adjusts the fundamental frequency automatically. Now you can read the distortion factor THD directly on the display.

- ***SPOT mode*** operation (knob SPOT/RANGE pressed) – in this mode the device allows automatic measurement of the distortions at three fixed frequencies (400Hz, 1KHz, 10kHz), without any necessity of manually adjusting the frequency of the notch filter (the frequency adjustment knob has no effect). The device measures the distortions for all the frequencies in the captured bandwidth of each of the three frequencies of the **SPOT** mode.

For both operating modes the selection of the measurement scale can be done as follows:

- **AUTO** mode (green knob not-pressed) – in this mode, the display range automatically changes according to the distortion factor. This behaviour is valid for voltmeter mode also.
- **HOLD** mode (green knob depressed) – the measurement range is not automatically changed. It remains set at the value set before the moment of the switching in **HOLD** mode.

2. Voltage Measurement

The device has the possibility to measure the RMS voltage of the (sinusoidal) input signal, using the left-side display. The voltmeter is not ***true RMS***. It will correctly measure RMS voltages only for sinusoidal signals.

<u>Measurements</u>

A. Measurement of the distortions for the signal generator

1. Measure the distortion factor for the sinusoidal signal generated by the signal generator that is available on your table (*the settings for the Distortion Meter are described in Annex A1; for the generator, in Annex A3*). The distortion factor will be measured for the frequencies f_1=500Hz and f_2=10KHz. Follow the steps:

- Generate a sinusoidal signal with the corresponding frequency. Input the signal to the distortion meter. Connect the **Y** output of the distortion meter to the input of the oscilloscope to visualise the signal obtained at the output of the notch filter (after eliminating the fundamental component). Prior to the tuning of frequencies (LED 13 or 14 lit), the fundamental component is not rejected. On the display, you can observe the sinusoidal waveform. After the tuning (neither LED is lit), the fundamental component is rejected and you can see an irregular waveform (harmonics + noise) on the display.
- The measurements will be done in **AUTO** display mode (the green button not pressed).
- With the **SPOT/RANGE** switch on **RANGE,** the three knobs [3] next to it have the meanings indicated above them: **x1, x10, x100** (just one knob can be pressed) and are responsible for the selection of the range of frequencies. The operating frequency results from combining the value of the pressed switch from [3] set, and the indication of the knob adjustment [15]. For example, $f = 200$ *Hz* can be obtained from the following combinations:
  - ○ indication [15] on 200 and **x1** pressed *OR*
  - ○ indication [15] on 20 and **x10** pressed

- Choose a combination as mentioned above, to obtain the corresponding frequency. Adjust knob [15] following the direction indicated by the red LED. When neither LED is lit, the device is within the capture bandwidth. The operation is completed automatically.
- Read THD on the display. The full-scale value is given by the yellow LED bellow the indication (100%, 30% ... 0.1%). Depending on that value, use one of the two scales on the indicator (full-scale = 1 or full-scale = 3) for reading. For example, if the yellow LED is on 0.3% and the pointer is on 2.5 on the full-scale = 3, the distortion factor THD is 0.25%.
- Read THD on the dB scale.

Explain: Does THD depend on frequency? Why?

2. Measure the distortion factor for sinusoidal, rectangular and triangular signals (change it from the generator). Measure THD for each signal using the frequency f_1 = 2 kHz. Measure in continuous mode (**SPOT/RANGE** knob on **RANGE**) and display mode **AUTO** (green switch not pressed).

Draw the signal shown on the scope connected to the Y output of the Distortion Meter (the notch filter is tuned on the fundamental frequency) when a rectangular signal is applied. Explain its shape.

3. Using the oscilloscope, visualise the sinusoidal and rectangular signals spectra.

Connect the oscilloscope in parallel with the generator (if you leave it on **Y** terminal, you will not see the fundamental component and the levels will not correspond, due to the automatic level control system!).

Set the oscilloscope FFT display mode using **MATH MENU** knob**,** then press **Operation** *softkey* till you select **FFT**. Set the horizontal deflection coefficient to 2,5 kHz/div. Draw the spectra of the two signals. For the sinusoidal signal, measure the fundamental component and noise levels, using the amplitude cursors (**Cursor → Type=Magnitude, Source →Math**). Using the frequency cursor (**Cursor → Type=Frequency**), measure the frequency of the fundamental component.

Remark* 1.** *To measure the fundamental component and the noise (for sinusoidal signal),* ***sample *acquisition* (**Acquire → Sample**) and infinite persistence (**Display → Persist=Infinite**) are recommended.

Remark* 2.** When *activating the cursors by pressing the* ***Cursor *knob, the 2 LEDs below the rotary knobs for the Y positions on CH1 and CH2 are lit.*

Using the amplitude and, respectively, the frequency cursors, measure for rectangular and triangular signals:

- the fundamental component level
- the level of the first three harmonics
- the frequency of the first three harmonics

What is the order of each of the three harmonics? (*k* in equations (4) and (5), notice the multiple of the fundamental frequency); notice that, for example, the second highest harmonic may not be for k=2, because some harmonics may be much lower than the others.

On the ground of these measurements, explain the results obtained at point 2.

***Remark* 3.** *To measure the fundamental component and the harmonics (of triangular and rectangular signals) Average acquisition* (**Acquire → Average** with **Averages=**32 or 64) and visualisation without persistence (**Display → Persist=off**) are recommended.

4. Measure the distortion degree of a sinusoidal signal having the frequency f=1kHz, for the following signal levels: 0dB, -20dB, -40dB. These levels will be adjusted from the generator and read on the voltmeter [11].

- Set the distortion meter on **SPOT** operating mode such that the three knobs [3] have the meaning according to the label below them, i.e., the three frequencies pre-set for this device. If the input signal frequency is according to the selected knob, the tuning is automatically done. The rotary adjustment [15] has no effect.
- Connect the oscilloscope in parallel on the input terminals of the distortion meter and visualise the signal spectrum (use **Math Menu → Operation FFT**). Place the horizontal cursors to measure the fundamental component level (for each input level required above) and the noise level. For the noise level, place the horizontal cursor approximately on its peak value.

Pay attention! *When reading on the voltmeter consider also the scale pointed out by the LED from group [8] (the 8 LEDs bellow pointer [11]). Two LEDs will light, one from the left set LEDs, one from the right set. Because the indication is in dB, the two corresponding values must be summed to the indication pointed by [11]. For example, if the needle shows -5dB and LEDs of 10dB and -60dB are lit, the obtained value is -55 dB.*

In which way THD (indicated by the device) depends on the level of the input signal? Why?

B. Measuring the distortions of a distributed-load amplifier

5. Measure the resistors (using the ohmmeter). Build the distributed-load amplifier in Fig. 2a on the solderless board. In Fig. 2b, you will find a suggestion for placing the components on the solderless board.

Using the DC voltmeter, verify that the operating voltage of the DC supply is around 6V; connect the DC supply wires on the horizontal rows found in the upper and lower side of the solderless board (VCC, GND). Use the digital multimeter on voltmeter mode (press **DCV** key).

Attention! *Verify, with the DC voltmeter, the polarity of the supply wires, before connecting them to the board. You should not connect the DC power supply with inverse polarity! You will be penalized if you burn the transistor due to incorrect connection!*

The transistor is a NPN general-purpose transistor (2N3904) in plastic capsule TO-92, with leads disposed as in Fig. 3.

a) Compute the bias point of the transistor (V_B, V_E, I_E, V_C), using the values measured for the resistors and for Vcc_{meas}. Then, measure these values using the DC voltmeter. Assume V_{BE}=0.7V and $\beta>100$.

b) Determine the amplification of the circuit: set the frequency from the generator to 3kHz and amplitude Uvv in=200mV. The output signal is seen on the oscilloscope, in the collector of the transistor (its waveform must be sinusoidal, as in Figure 4).

Simultaneously connect $v_{IN}(t)$ at CH1 of the scope and $v_{OUT}(t)$ on CH2, coupling AC (without DC bias) and Cy1,2 set such that the *peak-to-peak* signals can be viewed on 5-8 div on the display. For efficiency use ***Measure*** menu of the scope, set to indicate peak-to-peak values for CH1 and CH2 (***Measure →Source: CH1 → Value: Peak-Peak***, respectively ***Measure →Source: CH2 →Value: Peak-Peak***).

Remark: *The output signal has nonzero DC bias, because the amplifier is supplied between 0 and +6V, which implies that the output signal cannot have values lower than 0V. The DC bias is the value of V_C obtained at a). To obtain a zero DC bias for the output signal, you should have used two supplies (+V and –V).*

Which is the ideal value of V_C from the bias point to obtain the maximum output voltage excursion? **Explain**.

c) Measure the voltage amplification and the distortion factor of the sinusoidal signal at 3kHz, from the input and from the output of the amplifier for the following values of the input Uvv: 200mV, 1.1V, 1.5V, 2.2V. Use the ***Measure*** menu and point out the possible changes of shape in the output signal.

Pay attention! *Adjust CH1, CH2 as many times as needed such that the variation range for the signal be of 5-8div.*

Explain why does the factor of distortion vary for the output signals when compared to the minimum value among them.

d) Visualise in FFT mode and measure the spectral components of the output signal of the amplifier when its input is a 3kHz sinusoidal signal with Uvv: 200mV, 1.1V, 1.5V, 2.2V. Use ***Math Menu→ FFT→CH2*** with C_X = 2,5kHz, C_Y =10dB/div and acquisition ***Acquire→ Peak detect***.

Attention! *Prior to choosing the FFT mode, verify the coupling to be on AC for CH1 and CH2, and that Cy=2V/div.*

For each input value identify the fundamental component, measure its magnitude in dB, determine the number of harmonics and the noise level in dB.

Compute the amplitude as the ratio between the amplitudes of the output and input signals (the two amplitudes are read on the scope in divisions, as we are interested in their ratio).

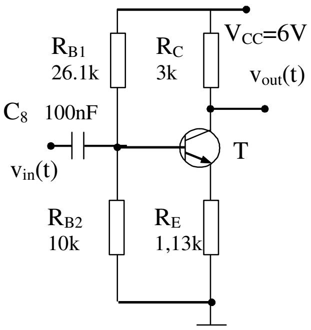


Fig. 2a) Electric schematics

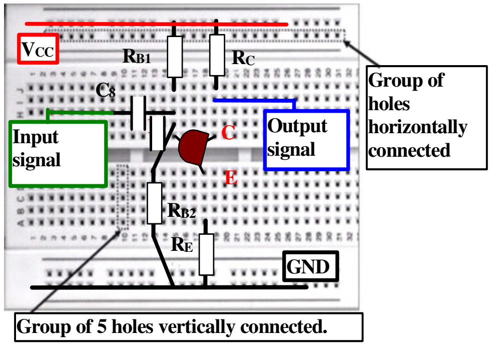


Fig. 2b) Connections on solderless board

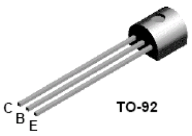
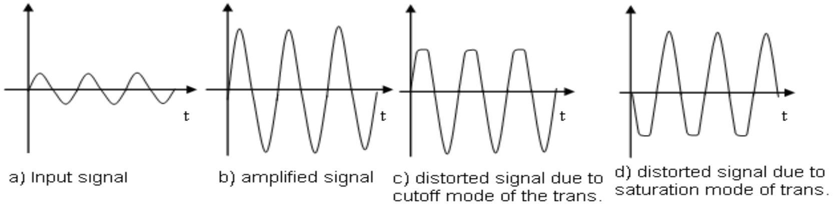


Fig. 3 Leads disposal for 2N3904 transistor.



a) Input signal

b) amplified signal

c) distorted signal due to cutoff mode of the trans.

d) distorted signal due to saturation mode of trans.

Fig. 4. Possible waveforms for the amplifier.

C. Determining the parameters of the notch filter (the filter that rejects the fundamental)

6. Determine the rejection filter parameters (the bandwidth of the notch filter and the attenuation characteristic). Ideally, the filter should attenuate *only* the frequency of 3kHz with -∞ dB. The other frequencies should have been attenuated with 0 dB (infinite sharp frequency characteristic).

Follow the steps:

- Generate a sinusoidal signal with the frequency f= 3kHz, having a level of 0dB, measured using the voltmeter [11].

- Tune the rejection filter of the distortion meter on the frequency of 3 kHz in **RANGE** operation mode. *From this moment on, you ought not to adjust the distortion meter anymore, especially its tuning frequency! Next, the frequency is adjusted from the generator!*

- Finely increase the frequency of the signal from the generator (select the digit corresponding to Hertz units, using the arrow keys below the rotary knob for the adjustment of the frequency from the generator, until the digit corresponding to Hz flashes; modify its value using the rotary knob).

At the beginning, one can notice that, by modifying finely the frequency of the signal from the generator, nothing happens with the distortion meter, as it is still in its capture bandwidth. In this situation, the distortion factor indicator remains at the minimum value. Write down the value for f_{up}, which is the highest frequency at which the distortion factor remains at its minimum value.

Also, write down this minimum value (by reading THD on dB scale; notice that the maximum value 0 dB corresponds to THD =100% and that, in this case, filter attenuation corresponds to the value measured for δ, but read in dB). When the frequency exceeds this value, distortion factor indication starts increasing. Write down the values of the frequencies set to generator for which the distortion factor, *expressed in dB*, reaches the values: -40dB, -30dB, -20dB, -10dB, -3dB.

- Repeat the measurements for the downwards variation of the frequency: start again at 3 kHz, slowly decrease the frequency and search for f_{low}. Up to this frequency, the distortion indication is constant. When you reach f_{low}, the indication will start varying. Determine the capture bandwidth. (see the Summary of theory section of this paper: Operating modes → 1. Measuring distortions).

- Plot the frequency characteristic of the filter using the results obtained above. The characteristic must be symmetrical with respect to the frequency f=3 kHz and will have the maximum attenuation, i.e., the minimum value in dB, at this frequency. Label the vertical axis with logarithmic values (in dB).

Questions and exercises:

1. Given the signal $s(t)=4\cdot\cos(\omega_0 t)+0.3\cdot\sin(3\omega_0 t)+0.4\cdot\cos(5\omega_0 t)$ [V], determine the theoretical distortion factor of this signal.
2. Decompose the signals in Fig. 5 in Harmonic Fourier Series.

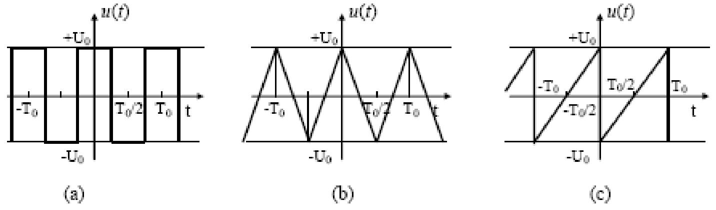


Fig. 5

3. Compute the theoretical distortion factor (THD) for all the signals in Fig. 5.

4. The signals in Fig. 5 are input to an ideal LPF with cutting frequency $f_{cut} = 4.5/T_0$. Compute the distortion factor THD for the output signals.

5. The sampling frequency of an oscilloscope is f_s=10MHz. N_x =10div. Which is the maximum frequency of a signal for which the FFT spectrum is correctly displayed?

6. Knowing that the frequency scale (when displaying FFT spectrum on the oscilloscope with N_x=10 div on OX) is linear, determine Cx_{max}, if the sampling frequency of the scope is $f_{sampling}$=100kHz.

7. If Cy=5dB/div, determine the maximum value of the ratio U_1/ U_2 that can be measured if N_y=8 div on OY (which displays the FFT of the signal).

8. The signal $u(t)=5\cdot\cos(\omega_0 t)$ [V] is the input of a nonlinear circuit with the transfer function $f(u)=u+0.1\cdot u^2$. Determine the distortion factor of the output signal, THD. (*Remark*: When computing δ, do not consider the DC bias). Which is the new value of THD for the output signal if the input signal becomes $s_1(t)=5\cdot\cos(\omega_0 t)+0.1\cdot\cos(4\omega_0 t)$ [V] ?

9. Which is the role of the rejection (notch) filter in distortion meter?

10. A signal has U_{RMS}=10V and THD = 100. Compute the fundamental component amplitude.

11. A signal with U_{RMS}=2V and U_{RMS} noise=10mV has SINAD=40dB. Compute THD.

12. Why does the distortion factor increase when the signal level is decreased, when measuring the distortions for the signal from the generator?

13. An ideal sinusoidal signal is applied at the input of an amplifier (with amplifying factor $a=1$). The output signal becomes distorted with a factor THD = 3% (this can be seen as an equivalent distortion factor with which the circuit distorts the input sinusoidal signal). Determine THD_2 of the signal at the output of the amplifier if, at the input, a sinusoidal signal with distortion factor THD_1 = 5% is applied.

14. An ideal sinusoidal signal is applied at the input of an amplifier (with amplifying factor $a=10$). The output signal becomes distorted with a factor THD_A = -40 dB. Determine THD_2 (in dB) of the signal at the output of the amplifier if, at the input, a sinusoidal signal with distortion factor THD_1 = -46 dB.is applied.

15. At the output of an amplifier (with amplifying factor $a=10$), a signal with a distortion factor THD_2 = -34 dB is obtained, when at the input a signal with THD_1 = -40 dB is applied. Determine the equivalent distortion factor introduced by the amplifier (THD_A).

16. Why do the distortions of the signal from the output of the amplifier increase, when the level of the input signal is increased?

17. For the setup in Fig. 6 (distributed-load), determine the maximum amplitude of the input signal such that the transistor does not become saturated (consider V_{BE} = 0.6V and $V_{CE\,sat}$ = 0.2V, β=50).

18. For the setup in Fig. 6 (distributed-load), determine the maximum amplitude of the input signal such that the transistor does not enter the cut-off working mode (consider V_{BE} = 0.6V and $V_{CE\,sat}$ = 0.2V, β=50).

19. For the setup in Fig. 6 (distributed-load), determine the maximum amplitude of the input signal such that the transistor works in normal active mode (consider V_{BE} = 0.6V and $V_{CE\,sat}$ = 0.2V, β=50).

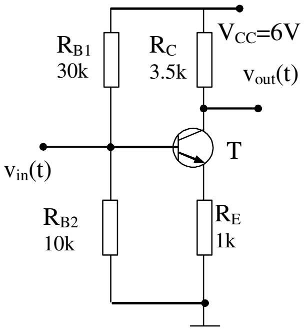


Fig 6. Distributed-load configuration.